West Virginia Mathematics

Grades 10, 11, 12 (All Courses)

Adopted 2024

Mathematical Habits of Mind

MHM1. Make sense of problems and persevere in solving them. MHM1

MHM2. Reason abstractly and quantitatively. MHM2

MHM3. Construct viable arguments and critique the reasoning of others. MHM3

MHM4. Model with mathematics. MHM4

MHM5. Use appropriate tools strategically. MHM5

MHM6. Attend to precision. MHM6

MHM7. Look for and make use of structure. MHM7

MHM8. Look for and express regularity in repeated reasoning. MHM8

Algebra I

Expressions and Equations

1. Interpret the structure of expressions and equations in terms of the context they model. A1.EE.1
    1. Interpret linear, exponential, and quadratic expressions that represent a quantity in terms of its context. M.A1HS.1
        a. Interpret parts of an expression, such as terms, factors, and coefficients. M.A1HS.1.A
        b. Interpret complicated expressions by viewing one or more of their parts as a single entity. M.A1HS.1.B
        c. Interpret the parameters in a linear function or exponential function of the form f(x) = a*b^x in terms of a context. M.A1HS.1.C
    2. Use the structure of quadratic and exponential expressions to identify ways to rewrite them. M.A1HS.2
2. Extend the properties of exponents to rational exponents. A1.EE.2
    3. Explain the connections between expressions with rational exponents and expressions with radicals using properties of exponents. Extend from application of properties of exponents for expressions with integer exponents. M.A1HS.3
    4. Rewrite expressions involving radicals, including simplifying, and rational exponents using the properties of exponents. M.A1HS.4
3. Write expressions in equivalent forms to solve problems. A1.EE.3
    5. Choose and produce an equivalent form of linear, exponential, and quadratic expressions to reveal and explain properties of the quantity represented by the expression through connections to a graphical representation of the function. M.A1HS.5
        a. Factor a quadratic expression to reveal the zeros of the function it defines. M.A1HS.5.A
        b. Complete the square in a quadratic expression, when a=1 only, to reveal the maximum or minimum value of the function it defines. M.A1HS.5.B
    c. Use the properties of exponents to transform expressions in exponential functions. For example, the expression 1.15^t can be rewritten as (1.15^{1/12})^{12t} ≈ 1.012^{12t} to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. M.A1HS.5.C
4. Perform arithmetic operations on polynomials. A1.EE.4
    6. Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Focus on linear or quadratic terms. M.A1HS.6
5. Create equations that describe numbers or relationships. A1.EE.5
    7. Create equations and inequalities in one variable, representing linear and exponential relationships, and use them to solve problems. In the case of exponential equations, limit to situations with integer inputs. M.A1HS.7
    8. Create equations in two or more variables, representing linear and exponential relationships between quantities. In the case of exponential equations, limit to situations with integer inputs. M.A1HS.8

9. Represent constraints by linear equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. M.A1HS.9

6. Solve equations and inequalities in one variable. A1.EE.6

    10. Solve linear equations including equations with coefficients represented by letters, simple exponential equations that rely on application of the laws of exponents, and compound linear inequalities in one variable. M.A1HS.10

    11. Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square when a=1 only, and the quadratic formula, as appropriate for the initial form of the equation. M.A1HS.11

        a. Recognize the concept of complex solutions when the quadratic formula gives complex solutions. M.A1HS.11.A

        b. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$. Derive the quadratic formula from this method of completing the square. M.A1HS.11.B

7. Solve systems of equations. A1.EE.7

    12. Analyze and solve pairs of simultaneous linear equations. M.A1HS.12

        a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. M.A1HS.12.A

        b. Solve simple cases by inspection (e.g., $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6). M.A1HS.12.B

        c. Solve real-word and mathematical problems leading to two linear equations in two variables (e.g., given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair). M.A1HS.12.C

    13. Understand and demonstrate ways to manipulate a system of two equations in two variables while preserving its solution set. M.A1HS.13

    14. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Include examples of solution sets with no solutions, an infinite number of solutions, and one solution. M.A1HS.14

    15. Solve a simple system consisting of a linear equation and a quadratic equation in two variables graphically. M.A1HS.15

8. Represent and solve equations and inequalities graphically. A1.EE.8

    16. Recognize that the graph of a linear or exponential equation in two variables is the set of all its solutions plotted in the coordinate plane. M.A1HS.16

    17. Explain why the x-coordinates of the points where the graphs of the linear and/or exponential equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations). M.A1HS.17

    18. Graph the solutions of a linear inequality in two variables as a half-plane and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. M.A1HS.18

Functions

1. Understand the concept of a function and use function notation. A1.F.1

    19. Use multiple representations of linear and exponential functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically. M.A1HS.19

    20. Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. M.A1HS.20

    21. Recognize arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (e.g., the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n)+ f(n-1) for n ≥ 1). M.A1HS.21

2. Interpret functions that arise in applications in terms of a context. A1.F.2

    22. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its linear, exponential, and quadratic graphs and, where applicable, to the quantitative relationship it describes. M.A1HS.22

        a. Key features of linear and exponential graphs include: intercepts; and intervals where the function is increasing, decreasing, positive, or negative. M.A1HS.22.A

        b. Key features of quadratic graphs include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximum or minimum; symmetry; and end behavior. M.A1HS.22.B

3. Analyze functions using different representations. A1.F.3

    23. Graph linear, exponential, and quadratic functions expressed symbolically and show key features of the graph. M.A1HS.23

        a. For linear functions, focus on intercepts. M.A1HS.23.A

        b. For exponential functions, focus on intercepts and end behavior. M.A1HS.23.B

        c. For quadratic functions, focus on intercepts, maxima, minima, end behavior, and the relationship between coefficients and roots to represent in factored form. M.A1HS.23.C

    24. Compare properties of two linear, exponential, or quadratic functions each represented in a different way, such as algebraically, graphically, numerically in tables, or from verbal descriptions. M.A1HS.24

    25. Write a function defined by a linear, exponential, or quadratic expression in different but equivalent forms to reveal and explain different properties of the function. M.A1HS.25

        a. Use the process of factoring and completing the square for a=1 only in a quadratic function to show zeros, extreme values, symmetry of the graph, the relationship between coefficients and roots represented in factored form and interpret these in terms of a context. M.A1HS.25.A

        b. Use the properties of exponents to interpret expressions in exponential functions. M.A1HS.25.B

4. Build a function that models a relationship between two quantities. A1.F.4

    26. Write linear, exponential, and quadratic functions that describe a relationship between two quantities. M.A1HS.26

        a. Determine an explicit expression, a recursive process, or steps for calculation from a context. M.A1HS.26.A

        b. Combine standard function types using arithmetic operations. M.A1HS.26.B

    27. Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship or given input-output pairs (include reading these from a table). M.A1HS.27

5. Build new functions from existing functions. A1.F.5

    28. Identify the effect on the graphs of linear and exponential functions, f(x), with f(x) + k, and the graphs of quadratic functions, g(x), with g(x) + k, k g(x), g(kx), and g(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. M.A1HS.28

6. Construct and compare linear, quadratic, and exponential models and solve problems. A1.F.6

    29. Distinguish between situations that can be modeled with linear functions, with exponential functions, and with quadratic functions. M.A1HS.29

        a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. M.A1HS.29.A

        b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. M.A1HS.29.B

        c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. M.A1HS.29.C

        d. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. Extend the comparison of linear and exponential growth to quadratic growth. M.A1HS.29.D

Geometry

1. Use coordinates to prove simple geometric theorems algebraically. A1.G.1

    30. Prove the slope criteria for parallel and perpendicular lines and use the slope criteria to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). M.A1HS.30

    31. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. M.A1HS.31

Statistics and Probability

1. Summarize, represent, and interpret data on a single count or measurement variable. A1.SP.1

    32. Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots). M.A1HS.32

    33. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets. M.A1HS.33

    34. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). M.A1HS.34

2. Summarize, represent, and interpret data on two categorical and quantitative variables. A1.SP.2

    35. Represent data on two quantitative variables on a scatter plot and describe how the variables are related. M.A1HS.35

        a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. M.A1HS.35.A

        b. Informally assess the fit of a function by plotting and analyzing residuals. Focus should be on situations for which linear models are appropriate. M.A1HS.35.B

        c. Fit a linear function for scatter plots that suggest a linear association. M.A1HS.35.C

3. Interpret linear models. A1.SP.3

    36. Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit. M.A1HS.36

    37. Distinguish between correlation and causation. M.A1HS.37

Geometry

Basics of Geometry

1. Experiment with transformations in the plane. G.BG.1
    1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. M.GHS.1
2. Identify and utilize inductive and deductive reasoning. G.BG.2
    2. Construct and justify the validity of a logical argument. M.GHS.2
        a. Identify the converse, inverse, and contrapositive of a conditional statement. M.GHS.2.A
        b. Translate a short, verbal argument into symbolic form. M.GHS.2.B
        c. Use Venn diagrams to represent set relationships. M.GHS.2.C
        d. Use inductive and deductive reasoning. M.GHS.2.D
3. Prove geometric theorems. G.BG.3
    3. Use appropriate methods of proof to prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent. M.GHS.3
4. Use coordinates to prove simple geometric theorems algebraically. G.BG.4
    4. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. M.GHS.4
5. Make geometric constructions. G.BG.5
    5. Make formal geometric constructions with a variety of tools and methods, such as a compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.: M.GHS.5
        a. copying a segment; M.GHS.5.A
        b. copying an angle; M.GHS.5.B
        c. bisecting a segment; M.GHS.5.C
        d. bisecting an angle; M.GHS.5.D
        e. constructing perpendicular lines, including the perpendicular bisector of a line segment; and M.GHS.5.E
        f. constructing a line parallel to a given line through a point not on the line. M.GHS.5.F

Transformations and Congruence

1. Experiment with transformations in the plane. G.TC.1
    6. Build on prior knowledge from rigid motions to: M.GHS.6
        a. represent transformations using geometric concepts in the plane. M.GHS.6.A
        b. describe transformations as functions that take points in the plane as inputs and give other points as outputs. M.GHS.6.B
        c. compare transformations that preserve distance and angle to those that do not. M.GHS.6.C
    7. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. M.GHS.7
    8. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. M.GHS.8
    9. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Describe a sequence of transformations that will carry a given figure onto another. M.GHS.9
2. Understand congruence in terms of rigid motions. G.TC.2
    10. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. M.GHS.10
    11. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. M.GHS.11
    12. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. M.GHS.12
    13. Use congruence criteria for triangles to solve problems and to prove relationships in geometric figures. M.GHS.13
3. Prove geometric theorems. G.TC.3
    14. Use appropriate methods of proof to prove theorems about triangles and lines. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. M.GHS.14
    15. Use appropriate methods of proof to prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. M.GHS.15
4. Use coordinates to prove simple geometric theorems algebraically. G.TC.4
    16. Use coordinates to prove simple geometric theorems about right triangles, quadrilaterals, and circles algebraically (e.g., derive the equation of a circle of given center and radius using the Pythagorean Theorem). M.GHS.16

Similarity and Trigonometry

1. Understand similarity in terms of similarity transformations. G.ST.1

    17. Verify experimentally the properties of dilations given by a center and a scale factor. M.GHS.17

        a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. M.GHS.17.A

        b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. M.GHS.17.B

    18. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. M.GHS.18

    19. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. M.GHS.19

2. Prove theorems involving similarity. G.ST.2

    20. Use appropriate methods of proof to prove theorems about triangles involving similarity. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. M.GHS.20

    21. Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Use the Pythagorean Theorem and similarity criteria to derive and apply special right triangles to solve problems. M.GHS.21

3. Define trigonometric ratios and solve problems involving right triangles. G.ST.3

    22. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. M.GHS.22

    23. Explain and use the relationship between the sine and cosine of complementary angles. M.GHS.23

    24. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. M.GHS.24

4. Apply trigonometry to general triangles. G.ST.4

    25. Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. M.GHS.25

    26. Prove the Laws of Sines and Cosines extending the definitions of sine and cosine to obtuse angles. M.GHS.26

    27. Understand and apply the Law of Sines and the Law of Cosines to solve problems and to find unknown measurements in right and non-right triangles. M.GHS.27

Circles

1. Understand and apply theorems about circles. G.C.1

    28. Prove that all circles are similar. M.GHS.28

    29. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. M.GHS.29

2. Find arc lengths and areas of sectors of circles. G.C.2

    30. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. M.GHS.30

3. Make geometric constructions. G.C.3

    31. Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle. M.GHS.31

    32. Construct a tangent line from a point outside a given circle to the circle. M.GHS.32

    33. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. M.GHS.33

Extending to Three Dimensions and Modeling

1. Explain volume formulas and use them to solve problems. G.E3D.1

    34. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. M.GHS.34

    35. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems, including how area and volume scale under similarity transformations. M.GHS.35

2. Visualize the relation between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations. G.E3D.2

    36. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. M.GHS.36

    37. Use two- and three-dimensional shapes and circles, their measures, and their properties to describe objects. M.GHS.37

        a. Apply concepts of density based on area and volume in modeling situations. M.GHS.37.A

        b. Apply geometric methods to solve design problems to satisfy given constraints. M.GHS.37.B

Statistics and Probability

1. Understand independence and conditional probability and use them to interpret data. G.SP.1
    38. Describe events as subsets of a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events. M.GHS.38
    39. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities. Use this characterization to determine if they are independent. M.GHS.39
    40. Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. M.GHS.40
    41. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. M.GHS.41
    42. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. M.GHS.42
2. Use the rules of probability to compute probabilities of compound events in a uniform probability model. G.SP.2
    43. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. M.GHS.43
    44. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model. M.GHS.44
    45. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B) and interpret the answer in terms of the model. M.GHS.45
    46. Use permutations and combinations to compute probabilities of compound events and solve problems. M.GHS.46
3. Use probability to evaluate outcomes of decisions. G.SP.3
    47. Use probabilities to make fair decisions. M.GHS.47
    48. Analyze decisions and strategies using probability concepts. M.GHS.48

Algebra II – Mathematics III

The Number System

1. Perform arithmetic operations with complex numbers. A2.NS.1
    1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b representing real numbers. M.A2HS.1
    2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. M.A2HS.2

Expressions and Equations

1. Use complex numbers in polynomial identities and equations. A2.EE.1
    3. Solve quadratic equations with real coefficients that have complex solutions. M.A2HS.3
    4. Factor special case polynomials with real coefficients that produce complex zeros. M.A2HS.4
    5. Show that the Fundamental Theorem of Algebra is true for quadratic polynomials with real coefficients. M.A2HS.5
2. Interpret the structure of expressions. A2.EE.2
    6. Interpret expressions including rational and polynomial expressions that represent a quantity in terms of its context. M.A2HS.6
        a. Interpret parts of an expression, such as terms, factors, and coefficients. M.A2HS.6.A
        b. Interpret complicated expressions by viewing one or more of their parts as a single entity. M.A2HS.6.B
    7. Use the structure of expressions including polynomial and rational expressions to identify ways to rewrite them. M.A2HS.7
3. Write expressions in equivalent forms to solve problems. A2.EE.3
    8. Derive the formula for the sum of a finite geometric and use the formula to solve problems. M.A2HS.8
4. Perform arithmetic operations on polynomials. A2.EE.4
    9. Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Perform operations on polynomials with degree higher than two. M.A2HS.9
5. Understand the relationship between zeros and factors of polynomials. A2.EE.5
    10. Apply the Remainder Theorem to polynomial functions. M.A2HS.10
    11. Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial. M.A2HS.11
6. Use polynomial identities to solve problems. A2.EE.6
    12. Prove polynomial identities and use them to describe numerical relationships. M.A2HS.12
    13. Apply the Binomial Theorem for the expansion of (x + y)ⁿ in powers of x and y for a positive integer n with coefficients determined, for example, by Pascal's Triangle. M.A2HS.13
7. Rewrite rational expressions. A2.EE.7
    14. Rewrite simple rational expressions in different forms; write a(x)/b(x) in different forms using inspection, long division, synthetic division, or, for the more complicated examples, a computer algebra system. M.A2HS.14
    15. Recognize that rational expressions form a system analogous to the rational numbers, namely, they are closed under addition, subtraction, multiplication, and

division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. M.A2HS.15

8. Understand solving equations as a process of reasoning and explain the reasoning. A2.EE.8

    16. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise M.A2HS.16

9. Represent and solve equations and inequalities graphically. A2.EE.9

    17. Explain why the x-coordinates of the points where the graphs of the linear, polynomial, rational, absolute value, exponential, and logarithmic equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). M.A2HS.17

10. Solve systems of equations. A2.EE.10

    18. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. M.A2HS.18

Functions

1. Create equations that describe numbers or relationships. A2.F.1

    19. Create equations and inequalities in one variable, representing linear, quadratic, simple rational, and exponential relationships, and use them to solve problems. M.A2HS.19

    20. Create equations in two or more variables, representing linear, exponential, and quadratic relationships, between quantities. M.A2HS.20

    21. Represent constraints by linear, exponential, or quadratic equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. M.A2HS.21

2. Interpret functions that arise in applications in terms of a context. A2.F.2

    22. Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its graph based on the behavior of data and context, and where applicable, to the quantitative relationship it describes. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; and end behavior. M.A2HS.22

    23. Select a model function based on behavior of data and context to calculate and interpret the average rate of change of linear, exponential, quadratic, and model functions based on behavior of data and context (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. M.A2HS.23

3. Analyze functions using different representations. A2.F.3

    24. Graph quadratic, polynomial, square root, cube root, piecewise-defined functions, including step functions and absolute value functions, exponential, and logarithmic functions expressed symbolically and show key features of the graph. Use applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. M.A2HS.24

        a. For polynomial functions, focus on identifying zeros and showing end behavior. M.A2HS.24.A

        b. For exponential and logarithmic functions, focus on showing intercepts and end behavior. M.A2HS.24.B

    25. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function focusing on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. M.A2HS.25

    26. Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions. Focus on applications and how key features relate to characteristics of a situation. M.A2HS.26

4. Build a function that models a relationship between two quantities. A2.F.4

    27. Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. M.A2HS.27

5. Build new functions from existing functions. A2.F.5

28. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types and use transformations to model situations. M.A2HS.28

29. Find inverse functions for simple polynomial, simple rational, simple radical, and use simple exponential functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. Consider situations where the domain of the function must be restricted in order for the inverse to exist. M.A2HS.29

6. Construct and compare linear, quadratic, and exponential models and solve problems. A2.F.6

30. For exponential models, express as a logarithm the solution to a•b^{ct} = d, where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. M.A2HS.30

Statistics and Probability

1. Summarize, represent, and interpret data on a single count or measurement variable. A2.SP.1

    31. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. M.A2HS.31

2. Understand and evaluate random processes underlying statistical experiments. A2.SP.2

    32. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. Compare theoretical and empirical results to evaluate the effectiveness. M.A2HS.32

    33. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. M.A2HS.33

3. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. A2.SP.3

    34. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. M.A2HS.34

    35. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error using simulation models for random sampling. Informally develop the concepts of statistical significance and variability. M.A2HS.35

    36. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Recognize that some unlikely results can occur solely through randomness inherent in the system and "statistical significance" represents this likelihood. Make use of statistics as a way of dealing with, not eliminating, this inherent randomness. M.A2HS.36

    37. Evaluate reports based on data. Focus on data collection and how conclusions can be drawn from data. M.A2HS.37

4. Use probability to evaluate outcomes of decisions. A2.SP.4

    38. Use probabilities to make fair decisions, including situations involving quality control, false positive, and false negative results. M.A2HS.38

    39. Analyze decisions and strategies using probability concepts, including situations involving quality control, false positive, and false negative results. M.A2HS.39

Trigonometry/Pre-calculus – Mathematics IV

Building Relationships among Complex Numbers, Vectors, and Matrices

1. Perform arithmetic operations with complex numbers. TPC.BR.1
    1. Find the conjugate of a complex number; use conjugates to find moduli (magnitude) and quotients of complex numbers. M.4HSTP.1
2. Represent complex numbers and their operations on the complex plane. TPC.BR.2
    2. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number. M.4HSTP.2
    3. Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. M.4HSTP.3
    4. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints. M.4HSTP.4
3. Represent and model with vector quantities. TPC.BR.3
    5. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes. M.4HSTP.5
    6. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. M.4HSTP.6
    7. Solve problems involving velocity and other quantities that can be represented by vectors. M.4HSTP.7
4. Perform operations on vectors. TPC.BR.4
    8. Add and subtract vectors. M.4HSTP.8
        a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. M.4HSTP.8.A
        b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. M.4HSTP.8.B
        c. Describe vector subtraction in terms of vector addition, represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. M.4HSTP.8.C
    9. Multiply a vector by a scalar. M.4HSTP.9
        a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. M.4HSTP.9.A
        b. Compute the magnitude of a scalar multiple of a vector. M.4HSTP.9.B
5. Perform operations on matrices and use matrices in applications. TPC.BR.5
    10. Use matrices to represent and manipulate data. M.4HSTP.10
    11. Multiply matrices by scalars to produce new matrices. M.4HSTP.11
    12. Add, subtract and multiply matrices of appropriate dimensions. M.4HSTP.12

13. Demonstrate that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. M.4HSTP.13
14. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. M.4HSTP.14
15. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. M.4HSTP.15
16. Work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. M.4HSTP.16

6. Solve systems of equations. TPC.BR.6

17. Represent a system of linear equations as a single matrix equation in a vector variable. M.4HSTP.17
18. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). M.4HSTP.18

Trigonometric and Inverse Trigonometric Functions of Real Numbers

1. Extend the domain of trigonometric functions using the unit circle. TPC.TIF.1

    19. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. M.4HSTP.19

    20. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. M.4HSTP.20

    21. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. M.4HSTP.21

    22. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. M.4HSTP.22

2. Model periodic phenomena with trigonometric functions. TPC.TIF.2

    23. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. M.4HSTP.23

    24. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. M.4HSTP.24

    25. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. M.4HSTP.25

    26. Solve multi-step trigonometric equations that require factoring or the use of identities. M.4HSTP.26

3. Prove and apply trigonometric identities. TPC.TIF.3

    27. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle. M.4HSTP.27

    28. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. M.4HSTP.28

Analysis and Synthesis of Functions

1. Interpret functions that arise in applications in terms of a context. TPC.ASF.1

    29. Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its graph based on the behavior of data and context, and, where applicable, to the quantitative relationship it describes.
Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity. M.4HSTP.29

2. Build a function that models a relationship between two quantities. TPC.ASF.2

    30. Write a function that describes a relationship between two quantities, including composition of functions. M.4HSTP.30

3. Analyze functions using different representations. TPC.ASF.3

    31. Graph trigonometric and rational functions expressed symbolically and show key features of the graph. M.4HSTP.31

        a. For trigonometric functions, focus on period, midline, amplitude, and phase shift. M.4HSTP.31.A

        b. For rational functions, focus on identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Analyze asymptotes and continuity informally using limits. M.4HSTP.31.B

4. Build new functions from existing functions. TPC.ASF.4

    32. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types and use transformations to model situations. M.4HSTP.32

    33. Find inverse functions. M.4HSTP.33

        a. Verify by composition that one function is the inverse of another. M.4HSTP.33.A

        b. Read values of an inverse function from a graph or a table, given that the function has an inverse. Compute values of inverse functions from graphs and recognize the graph of an inverse function is the graph of the original function reflected about y=x. M.4HSTP.33.B

        c. Produce an invertible function from a non-invertible function by restricting the domain. M.4HSTP.33.C

    34. Use an understanding of the inverse relationship between exponents and logarithmic functions to: M.4HSTP.34

        a. Graph logarithms, M.4HSTP.34.A

        b. Derive properties of logarithms, and M.4HSTP.34.B

        c. Use these properties to model and solve problems and applications involving exponential and logarithmic functions. M.4HSTP.34.C

Derivations in Analytic Geometry

1. Use conic sections to solve applications. TPC.DAG.1
    35. Derive the equations of a parabola, circle, ellipses, and hyperbolas using their key components. Graph the equations of these conic sections.
Key components include asymptotes, center, directrix, foci, and radius. M.4HSTP.35
    36. Solve problems and applications that model conic sections. M.4HSTP.36

Series and Informal Limits

1. Use sigma notations to evaluate finite sums. TPC.S.1
    37. Develop sigma notation and use it to write series in equivalent form. M.4HSTP.37
    38. Apply the method of mathematical induction to prove summation formulas. For example, verify the sum of squares formula. M.4HSTP.38
2. Extend geometric series to infinite geometric series. TPC.S.2
    39. Develop intuitively that the sum of an infinite series of positive numbers can converge and derive the formula for the sum of an infinite geometric series. Apply infinite geometric series models intuitively. M.4HSTP.39

Applied Statistics

Exploring Data

1. Select appropriate graphical and numerical methods to explore data. AS.ED.1
    1. Generate appropriate ways to display various types of data. M.ASHS.1
    2. Calculate appropriate measures of center, variability, and position for data. M.ASHS.2
    3. Use graphical displays and summary statistics to make conclusions. Informally develop the concept of statistical significance; a result that is unlikely to have occurred by chance alone. M.ASHS.3
    4. Represent data in two variables to model relationships between quantities. M.ASHS.4
    5. Select a function that models a relationship between two quantities and interpret key features of graphs and tables in terms of the quantities. M.ASHS.5
    6. Compare characteristics of two data sets each represented in different ways (algebraically, graphically, numerically, and verbally). M.ASHS.6
    7. Use appropriate measures of center and spread to describe a distribution. M.ASHS.7

Designing Studies

1. Design and implement a plan to collect and analyze data. AS.DS.1

    8. Develop a process for making inferences about population parameters based on a random sample through data collection and analysis. M.ASHS.8

    9. Evaluate the results from a given data-generating process to determine consistency between theoretical and experimental probabilities. M.ASHS.9

    10. Recognize the purposes of and differences among sample surveys, experiments, and observational studies. Explain the importance of randomization in each method. M.ASHS.10

    11. Use data from a sample survey to estimate a population mean or proportion. M.ASHS.11

    12. Design and conduct an experiment to compare two treatments. M.ASHS.12

Functions and Modeling

1. Explore expressions, functions, and models to describe numbers or relationships. AS.FM.1
    13. Create equations and inequalities in one variable, representing linear, exponential, quadratic, and simple rational functions, and use them to solve problems. M.ASHS.13
    14. Develop the concept of a complex number i such that $i^2 = -1$. Understand that every complex number can be written in the form a + bi with a and b real. M.ASHS.14
    15. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. M.ASHS.15
    16. Use the structure of polynomial and rational expressions to identify ways to rewrite them. M.ASHS.16
    17. Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial. M.ASHS.17
    18. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. M.ASHS.18
    19. Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise. M.ASHS.19
    20. Rearrange formulas to highlight a quantity of interest, using the same reasoning as ir solving equations (e.g., solve for σ and Margin of Error = for n). M.ASHS.20
    21. Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative extrema; symmetries; and end behavior. M.ASHS.21
2. Use probability to evaluate outcomes and make decisions. AS.FM.2
    22. Connect sampling variability and margin of error to generate and interpret plausible parameter values. M.ASHS.22
    23. Interpret results from a randomized experiment comparing two treatments. Use simulations to decide if experimental results are significant. M.ASHS.23
    24. Evaluate claims based on data reports. M.ASHS.24
    25. Use probability rules to make fair decisions. M.ASHS.25
    26. Use two-way tables, tree diagrams, Venn diagrams, or 10 x 10 grids to model probabilities. M.ASHS.26
    27. Justify a decision using probability rules (e.g., product testing, medical testing, weather forecasting, marketing, or sports coaching decisions). M.ASHS.27

28. Perform appropriate calculations for given outcomes and decisions based on expected values for non-normal distributions. M.ASHS.28

29. Given data from a normal distribution, use the mean and standard deviation to estimate population percentages. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Recognize that there are data sets for which such a procedure is not appropriate. M.ASHS.29

Transition Mathematics for Seniors

Number and Quantity – The Real Number System

1. Reason quantitatively and use units to solve problems. TM.NQR.1

    1. Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. M.TMS.1

    2. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. M.TMS.2

Number and Quantity – The Complex Number System

1. Use complex numbers in polynomial identities and equations. TM.NQC.1

    3. Solve quadratic equations with real coefficients that have complex solutions. M.TMS.3

Algebra – Seeing Structure in Expressions

1. Interpret the structure of expressions. TM.AS.1

    4. Use the structure of quadratic and exponential expressions to identify ways to rewrite them. M.TMS.4

2. Write expressions in equivalent forms to solve problems. TM.AS.2

    5. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. M.TMS.5

    a. Factor a quadratic expression to reveal the zeros of the function it defines. M.TMS.5.A

    b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. M.TMS.5.B

3. Understand the connections between proportional relationship, lines, and linear equations. TM.AS.3

    6. Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways. M.TMS.6

    7. Explain (e.g., using similar triangles) why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. M.TMS.7

    8. Solve linear equations in one variable. M.TMS.8

Algebra – Arithmetic with Polynomials and Rational Expressions

1. Perform arithmetic operations on polynomials. TM.AP.1

    9. Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials. M.TMS.9

Algebra – Creating Equations

1. Create equations that describe numbers or relationships. TM.ACE.1

    10. Create equations and inequalities in one variable, representing linear, exponential, quadratic, and simple rational relationships, and use them to solve problems. M.TMS.10

    11. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. M.TMS.11

    12. Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. M.TMS.12

    13. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. M.TMS.13

Algebra – Reasoning with Equations and Inequalities

1. Understand solving equations as a process of reasoning and explain the reasoning. TM.ARE.1

    14. Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise. M.TMS.14

2. Solve equations and inequalities in one variable. TM.ARE.2

    15. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. M.TMS.15

    16. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. M.TMS.16

    17. Solve quadratic equations in one variable by inspection (e.g., $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation. M.TMS.17

        a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. M.TMS.17.A

        b. Recognize the concept of complex solutions when the quadratic formula gives complex solutions, and write them as a ± bi for real numbers a and b. M.TMS.17.B

3. Solve systems of equations. TM.ARE.3

    18. Understand and demonstrate ways to manipulate a system of two equations in two variables while preserving its solution set. M.TMS.18

    19. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. M.TMS.19

    20. Explain why the x-coordinates of the points where the graphs of the linear, polynomial, rational, absolute value, and exponential equations y = f(x) and y = g(x) intersect are the solution of the equation f(x) = g (x); find the solution approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). M.TMS.20

4. Represent and solve equations and inequalities graphically. TM.ARE.4

    21. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. M.TMS.21

    22. Graph the solutions to a linear inequality in two variables as a half-plane graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. M.TMS.22

Functions – Interpreting Functions

1. Understand the concept of a function and use function notation. TM.F.1

    23. Use multiple representations of linear and exponential functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically. M.TMS.23

2. Interpret functions that arise in applications in terms of the context. TM.F.2

    24. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. M.TMS.24

    25. Interpret the parameters in a linear or exponential function in terms of a context. M.TMS.25

    26. Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity. M.TMS.26

    27. Distinguish between situations that can be modeled with linear functions and with exponential functions. M.TMS.27

3. Analyze functions using different representations. TM.F.3

    28. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. M.TMS.28

    29. Describe qualitatively the functional relationship between two quantities by analyzing a graph. M.TMS.29

    30. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. M.TMS.30

    31. Graph linear, quadratic, and polynomial functions expressed symbolically and show key features of the graph. M.TMS.31

        a. For linear functions, focus intercepts. M.TMS.31.A

        b. For quadratic functions, focus on intercepts, maxima, minima, end behavior, and the relationship between coefficients and roots to represent in factored form. M.TMS.31.B

        c. For polynomial functions, focus on identifying zeros and showing end behavior. M.TMS.31.C

    32. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasingly linearly, quadratically, or (more generally) as a polynomial function. M.TMS.32

    33. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. M.TMS.33

34. Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions. M.TMS.34

Functions - Building Functions

1. Build a function that models a relationship between two quantities. TM.FB.1

    35. Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship, or given input-output pairs (include reading these from a table). M.TMS.35

    36. Write a function that describes a relationship between two quantities. M.TMS.36

        a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. M.TMS.36.A

        b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. M.TMS.36.B

Geometry – Geometric Measuring and Dimension

1. Explain volume formulas and use them to solve problems. TM.GGM.1

    37. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. M.TMS.37

    38. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. M.TMS.38

Geometry – Expressing Geometric Properties with Equations

1. Use coordinates to prove simple geometric theorems algebraically. TM.EG.1

    39. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). M.TMS.39

    40. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. M.TMS.40

Geometry – Modeling with Geometry

1. Apply geometric concepts in modeling situations. TM.MG.1

    41. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios). M.TMS.41

Statistics and Probability - Interpreting Categorical & Quantitative Data

1. Summarize, represent, and interpret data on two categorical and quantitative variables. TM.SP.1
    42. Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Interpret linear models. M.TMS.42
    43. Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit. M.TMS.43
    44. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. M.TMS.44
    45. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. M.TMS.45
2. Summarize, represent, and interpret data on a single count or measurement variable. TM.SP.2
    46. Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots). M.TMS.46
    47. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets. M.TMS.47
    48. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). M.TMS.48
    49. Distinguish between correlation and causation. M.TMS.49
3. Understand and evaluate random processes underlying statistical experiments. TM.SP.3
    50. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. M.TMS.50

Advanced Mathematical Modeling

Developing College and Career Skills

1. Mathematics as a language. AM.M.1
    1. Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others. M.AMM.1
    2. Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems. M.AMM.2
2. Tools for problem solving. AM.M.2
    3. Gather data, conduct investigations, and apply mathematical concepts and models to solve problems in mathematics and other disciplines. M.AMM.3

Finance

1. Understand financial models. AM.F.1

    4. Determine, represent, and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., business loans, auto, mortgage, and/or credit card). M.AMM.4

    5. Determine, represent, and analyze mathematical models for investments involving simple and compound interest with and without additional deposits (e.g., savings accounts, bonds, and/or certificates of deposit). M.AMM.5

    6. Determine, represent, and analyze mathematical models for inflation and the Consumer Price Index using concepts of rate of change and percentage growth. M.AMM.6

2. Personal use of finance. AM.F.2

    7. Research and analyze personal budgets based on given parameters (e.g., fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission, career choice, geographic region, retirement and/or investment planning). M.AMM.7

    8. Research and analyze taxes including payroll, sales, personal property, real estate, and income tax returns. M.AMM.8

Probability

1. Analyze information using probability and counting. AM.P.1

    9. Use the Fundamental Counting Principle, permutations, and combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers. M.AMM.9

    10. Determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables and area models, to make decisions in problem situations. M.AMM.10

2. Manage uncertainty. AM.P.2

    11. Use probabilities to make and justify decisions about risks in everyday life. M.AMM.11

    12. Calculate expected value to analyze mathematical fairness, payoff and risk. M.AMM.12

Statistics

1. Critique statistics. AM.S.1
    13. Identify limitations or lack of information in studies reporting statistical information, especially when studies are reported in condensed form. M.AMM.13
    14. Interpret and compare the results of polls, given a margin of error. M.AMM.14
    15. Identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions, including assertions of cause and effect versus correlation. M.AMM.15
    16. Describe strengths and weaknesses of sampling techniques, data and graphical displays and interpretations of summary statistics, and other results appearing in a study, including reports published in the media. M.AMM.16
2. Perform statistical analysis. AM.S.2
    17. Identify the population of interest, select an appropriate sampling technique, and collect data. M.AMM.17
    18. Identify the variables to be used in a study. M.AMM.18
    19. Determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results. M.AMM.19
    20. Create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features. M.AMM.20
    21. Determine possible sources of variability of data, both those that can be controlled and those that cannot be controlled. M.AMM.21
3. Communicate statistical information. AM.S.3
    22. Report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied. M.AMM.22
    23. Communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language. M.AMM.23

Modeling

1. Manage numerical data. AM.M.1

    24. Solve problems involving large quantities that are not easily measured. M.AMM.24

    25. Use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems. M.AMM.25

2. Model data and change with functions. AM.M.2

    26. Determine or analyze an appropriate model for problem situations - including linear, quadratic, power, exponential, logarithmic and logistic functions (e.g., stopping distance, period of a pendulum, population growth, Richter Scale, and/or Fujita Tornado Scale). M.AMM.26

    27. Determine or analyze an appropriate cyclical model for problem situations that can be modeled with trigonometric functions (e.g., predator-prey models, tide heights, diurnal cycle, and/or music). M.AMM.27

    28. Determine or analyze an appropriate piecewise model for problem situations (e.g., postal rates, phase change graphs, sales tax, and/or utility usage rates). M.AMM.28

    29. Solve problems using recursion or iteration (e.g., fractals, compound interest, population growth or decline, and/or radioactive decay). M.AMM.29

    30. Collect numerical bivariate data; use the data to create a scatter plot; determine whether or not a relationship exists; if so, select a function to model the data, justify the selection and use the model to make predictions. M.AMM.30

Networks

1. Network for decision making. AM.N.1

    31. Solve problems involving scheduling or routing situations that can be represented by a vertex-edge graph; find critical paths, Euler paths, Hamiltonian paths, and minimal spanning trees (e.g., Konigsberg bridge problem, mail vs. Fed Ex delivery routes, Kolam drawings of India, traveling salesman problem, and/or map coloring). M.AMM.31

    32. Construct, analyze, and interpret flow charts in order to develop and describe problem solving procedures. M.AMM.32

Social Decision Making

1. Make decisions using ranking and voting. AM.SD.1

    33. Apply and analyze various ranking algorithms to determine an appropriate method for a given situation (e.g., fair division, apportionment, and/or search engine results). M.AMM.33

    34. Analyze various voting and selection processes to determine an appropriate method for a given situation (e.g., preferential vs. non-preferential methods, and/or weighted voting). M.AMM.34

Geometry

1. Concrete geometric representation (physical modeling). AM.G.1
    35. Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications. M.AMM.35
    36. Solve geometric problems involving inaccessible distances. M.AMM.36
2. Abstract geometric representation (matrix modeling). AM.G.2
    37. Use vectors to represent and solve applied problems. M.AMM.37
    38. Use matrices to represent geometric transformations and solve applied problems. M.AMM.38

Calculus

Algebra

1. Understand the key concepts, connections and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways. C.A.1

    1. Use abstract notation to apply properties of algebraic, trigonometric, exponential, logarithmic, and composite functions, as well as their inverses, represented graphically, numerically, analytically, and verbally; and demonstrate an understandin of the connections among these representations. M.C.1
    2. Demonstrate a conceptual understanding of the definition of a limit via the analysis of continuous and discontinuous functions represented using multiple representations (e.g., graphs and tables). M.C.2
    3. Use the properties of limits including addition, product, quotient, composition, and squeeze/sandwich theorem to calculate the various forms of limits: one-sided limits, limits at infinity, infinite limits, limits that do not exist, and special limits such as and M.C.3
    4. Apply the definition of continuity to determine where a function is continuous or discontinuous including continuity at a point, continuity over an interval, application of the Intermediate Value Theorem, and graphical interpretation of continuity and discontinuity. M.C.4
    5. Investigate and apply the definition of the derivative graphically, numerically, and analytically at a point; conceptually interpret the derivative as an instantaneous rate c change and the slope of the tangent line. M.C.5
    6. Discriminate between the average rate of change and the instantaneous rate of chang using real-world problems. M.C.6
    7. Recognize when the Extreme Value Theorem indicates that function extrema exist. M.C.7
    8. Quickly recall and apply rules of differentiation including the constant multiple rule, the sum rule, the difference rule, the product rule, the quotient rule, the power rule, and the chain rule as applied to algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions using techniques of both explicit and implicit differentiation. M.C.8
    9. Apply Rolle's Theorem and the Mean Value Theorem to real-world problems. M.C.9
    10. Construct and use mathematical models to solve optimization, related-rates, velocity and acceleration problems. M.C.10
    11. Determine antiderivatives that follow from derivatives of basic functions and apply substitution of variables. M.C.11
    12. Evaluate definite integrals using basic integration properties such as addition, subtraction, constant multipliers, the power rule, substitution, and change of limits. M.C.12
    13. Characterize the definite integral as the total change of a function over an interval and use this to solve real-world problems. M.C.13
    14. Apply the Fundamental Theorem of Calculus to evaluate definite integrals and to formulate a cumulative area function and interpret the function as it relates to the

integrand. M.C.14

15. Use limits to deduce asymptotic behavior of the graph of a function. M.C.15
16. Compare and contrast the limit definition (not delta epsilon) of continuity and the graphical interpretation of the continuity of a function at a point; recognize different types of discontinuities. M.C.16
17. Develop tangent lines as best linear approximations to functions near specific points explain this conceptually; construct these tangent lines; and apply this concept to Newton's Method. M.C.17
18. Investigate and explain the relationships among the graphs of a function, its derivative and its second derivative; construct the graph of a function using the first and second derivatives including extrema, points of inflection, and asymptotic behavior. M.C.18
19. Approximate areas under a curve using Riemann sums by applying and comparing left, right, and midpoint methods for a finite number of subintervals. M.C.19

Geometry

1. Apply the key concepts, connections and applications of limits, continuity, derivatives, and integration for a wide variety of regions. C.G.1

20. Justify why differentiability implies continuity and classify functional cases when continuity does not imply differentiability. M.C.20
21. Calculate a definite integral using Riemann sums by evaluating an infinite limit of a sum using summation notation and rules for summation. M.C.21
22. Use integration to solve problems that involve linear displacement, total distance, position, velocity, acceleration, and area between curves by looking at both functions of x and functions of y; utilize units to interpret the physical nature of the calculus process. M.C.22

Data Analysis and Probability

1. Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data. C.DA.1

23. Identify a real-world situation that involves quantities that change over time; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous function that describes the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically, and verbally using the predictive and analytic tools of calculus. M.C.23

Financial Algebra/Mathematics

Mathematical Language in a Financial Context

1. Communicate reasoning and decisions. FA.A.1
    1. Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical decision making (e.g., demonstrate reasoning skills in creating and presenting a budget of monthly expenses based on a career pathway income, and analyze the soundness of the mathematical reasoning of others; determine outlook for a chosen career pathway and use the average salary to determine if the desired cost of living can be met). M.FAM.1
    2. Communicate with and about mathematics in a financial context. M.FAM.2
    3. Communicate with and about mathematics in writing and orally, both independently and collaboratively, by preparing financial plans (e.g., plan for an emergency savings fund that will last three to six months in the case of loss of income; determine the total percentage of income paid to taxes or the percentage of total salary that a benefits package represents). M.FAM.3

Algebra/Mathematics of Finance

2. Use algebraic reasoning and techniques. FA.A.2

    4. Interpret parts of an expression or equation, such as terms, factors, and coefficients, in a variety of financial models including those found in stock markets, automobile financing, and in banking contexts. M.FAM.4

    5. Create and solve linear equations and inequalities in one variable and use them to solve problems in financial applications that may include, but are not limited to, stock markets, automobile ownership, business modeling, or employment (e.g., calculate wages by hourly rates or pay periods to make decisions about pay in a real-world context). M.FAM.5

    6. Create equations in two or more variables to represent relationships between quantities in a financial context; graph equations on coordinate axes with labels and scales. Financial contexts may include, but are not limited to, stock markets, automobile ownership, business modeling employment, banking, consumer debt, and independent living decisions regarding taxes or planning for retirement (e.g., create a linear expense equation based on fixed and variable expenses and graph choosing an appropriate scale and origin for the graph). M.FAM.6

    7. Represent constraints in financial applications by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context (e.g., create a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental; find the percentage of total salary that a benefits package represents; calculate taxes owed based on a given income and tax table and determine total percentage of income paid to taxes; calculate the gross pay and net pay using the FICA percentage (7.65%), retirement contribution, and worker's compensation insurance (employer match)). M.FAM.7

    8. Rearrange formulas for financial applications to highlight a quantity of interest, using the same reasoning as in solving equations. Know difference between growth and decay functions (e.g., solve the literal equation for exponential depreciation to find a depreciation rate and the literal equation for continuous interest to find the interest rate; apply the formula for average daily balance, (average daily balance*APR*days in billing cycle)/365, using literal equations with varying APRs and billing cycles). M.FAM.8

    9. Solve systems of linear equations exactly and approximately (with graphs) in making financial decisions, focusing on pairs of linear equations in two variables (e.g., create and solve a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental). M.FAM.9

    10. Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials (e.g., combine the polynomials that model income and expense to create a profit model). M.FAM.10

    11. Solve quadratic equations in one variable in a financial context that may include, but are not limited to, business modeling or employment decisions (e.g., given a quadratic equation that models a profit function, determine the break-even points; apply braking distance/stopping distance formulas to solve problems related to driving and safety data). M.FAM.11

Financial Modeling with Functions

3. Construct, graph, use, and interpret functions. FA.A.3

    12. Use functions to model financial situations. Use multiple representations of functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically (e.g., develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; use linear and polynomial functions to evaluate and communicate quantities as required by Internal Revenue Service and Social Security Administration regulations and to determine when and why the models may be discontinuous). M.FAM.12

    13. Use function notation in financial applications, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a financial context (e.g., in making decisions regarding retirement income, apply the formula A(t) = Pe^{rt} to determine future value). M.FAM.13

    14. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Use this relationship in analyzing financial situations (e.g., compare the linear function modeling simple interest with the exponential function modeling compound interest). M.FAM.14

    15. Select a function that models a relationship between two quantities in financial contexts, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship (e.g., write, graph, and interpret the revenue (quadratic) function in comparison to the expense (linear) function using key features of the functions; reason quantitatively to compare subsidized and unsubsidized loans, as well as other forms of financial aid available to college students; calculate mortgage payments, reasoning and making decisions about the length of the loan and a fixed versus adjustable rate mortgage). M.FAM.15

    16. Interpret the parameters in a linear or exponential function in terms of a context (e.g., investigate and compare, using technology and regression, historical data to determine if automobile depreciation follows a linear or exponential model). M.FAM.16

    17. Construct linear and exponential functions modeling financial contexts, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship, or given input-output pairs including reading these from a table (e.g., utilize linear and exponential functions to compare simple with compound interest). M.FAM.17

    18. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. Data may address, but is not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning. M.FAM.18

    19. Calculate and interpret the average rate of change of a function modeling a financial context (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph (e.g., examine depreciation trends). M.FAM.19

20. Graph functions expressed symbolically and show key features of the graph (e.g., graph the linear, quadratic, or exponential curve that models the demand versus supply functions and find the equilibrium point with and without technology). M.FAM.20

21. Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions (e.g., utilize linear and exponential functions to compare simple with compound interest; calculate and compare using both the loan payment formula and payment schedules in table format, the monthly cost of purchasing an automobile, and discuss the feasibility of that payment in relation to monthly budget; compare two functions showing interest accrued when paying the minimum monthly payment over time compared to paying a larger monthly payment; identify and compare the average rate of change between given time periods). M.FAM.21

22. Graph linear and quadratic functions and show intercepts, maxima, and minima (e.g., in the model of a profit function, determine the break-even points, the maximum possible loss, and the maximum profit). M.FAM.22

23. Write a function that describes a relationship between two quantities in a financial context (e.g., calculate the costs associated with purchasing a vehicle, including leasing, purchasing with cash, or with a loan). M.FAM.23

24. Identify the effect on functions that model financial situations of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology (e.g., identify the impact of a change in a constraint in a function that models retirement planning, business income and expenses, or employment benefits). M.FAM.24

25. Graph square root, cube root, and piecewise-defined functions that model financial situations, including step functions and absolute value functions (e.g., develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; analyze graphs of functions that model profit). M.FAM.25

26. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model financial situations, and translate between the two forms (e.g., create recursive and explicit models of sequences related to retirement planning, amortization schedules for a loan, comparing subsidized and unsubsidized loans, reasoning and making decisions about the length of the loan and a fixed versus adjustable rate mortgage). M.FAM.26

27. Apply exponential formulas to solve for future and present value of investments by hand or with graphing technology (e.g., PV=FV*(1/(1+r)ⁿ) and A(t) = Pe^{rt}). M.FAM.27

Financial Modeling with Data

4. Represent, summarize, and evaluate data. FA.A.4

    28. Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots). M.FAM.28

    29. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit regression lines to scatterplots and make predictions based on lines of best fit. Find and interpret correlation coefficients of regression equations in financial situations (e.g., use scatter plots to show correlation between two funds, two stocks, a stock and the general market, or in business situations to forecast sales or to compare revenue to the number of units sold). M.FAM.29

    30. Create a data display modeling financial situations. M.FAM.30

    31. Summarize categorical data in various forms (e.g., two-way frequency tables, circle graphs, segmented bar charts). Interpret relative frequencies in the context of the data in making financial decisions. M.FAM.31

    32. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). M.FAM.32

    33. Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays (e.g., use units appropriately as a way to understand multi-step problems in relationship to understanding credit card fees and finance charges; compute distance, rate and time to solve problems to analyze driving and safety data, using single and multiple unit conversion; use and compare researched reaction times and vehicle velocity, as well as accepted equations to solve problems with braking distances). M.FAM.33

    34. Use financial models from automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning to solve problems. M.FAM.34

    35. Evaluate reports based on data. Data may address, but is not limited to, planning for retirement or stock markets. M.FAM.35

    36. Use probability and expected value to analyze financial situations (e.g., model and compare automobile insurance policies). M.FAM.36

    37. Evaluate the impact of taxes on business ownership including property tax, sales tax, social security, retirement, and disability benefits. Evaluate the impact of taxes on personal finance decisions. M.FAM.37

Mathematics – Quantitative Reasoning

Logical Reasoning

1. Mathematics as a language. MQR.L.1
    1. Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others. M.QR.1
    2. Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems. M.QR.2
2. Tools for problem solving. MQR.L.2
    3. Gather data, conduct investigations, and apply mathematical concepts and models to solve problems in mathematics and other disciplines. M.QR.3

Algebraic Modeling

1. Understand ratio concepts and use ratio reasoning to solve problems. MQR.A.1

    4. Use ratio and rate reasoning to solve real-world and mathematical problems. M.QR.4

        a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. M.QR.4.A

        b. Solve unit rate problems including those involving unit pricing and constant speed (e.g., if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?). M.QR.4.B

        c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. M.QR.4.C

        d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. M.QR.4.D

2. Work with integer exponents, scientific notation, and radicals. MQR.A.2

    5. Know and apply the properties of integer exponents to generate equivalent numerical expressions. M.QR.5

    6. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. M.QR.6

    7. Rewrite expressions involving radicals and rational exponents using the properties of exponents. M.QR.7

3. Reason quantitatively and use units to solve problems. MQR.A.3

    8. Define appropriate quantities for the purpose of descriptive modeling. M.QR.8

4. Represent and solve equations and inequalities graphically. MQR.A.4

    9. Recognize that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. M.QR.9

5. Explain volume formulas and use them to solve problems. MQR.A.5

    10. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. M.QR.10

6. Understand financial models. MQR.A.6

    11. Determine, represent and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., business loans, auto, mortgage, and/or credit card). M.QR.11

    12. Determine, represent and analyze mathematical models for investments involving simple and compound interest with and without additional deposits (e.g., savings accounts, bonds, and/or certificates of deposit). M.QR.12

    13. Research and analyze taxes including payroll, sales, personal property, real estate, and income tax returns. M.QR.13

7. Reason quantitatively and use units to solve problems. MQR.A.7

14. Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. M.QR.14

15. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. M.QR.15

8. Create equations that describe numbers or relationships. MQR.A.8

16. Create equations and inequalities in one variable, representing linear, quadratic, simple rational, and exponential relationships, and use them to solve problems. M.QR.16

17. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. M.QR.17

9. Construct and compare linear, quadratic, and exponential models and solve problems. MQR.A.9

18. Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship, or given input-output pairs (include reading these from a table). M.QR.18

10. Build a function that models a relationship between two quantities. MQR.A.10

19. Write a function that describes a relationship between two quantities. M.QR.19

    a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. M.QR.19.A

    b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. M.QR.19.B

11. Interpret linear models. MQR.A.11

20. Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit. M.QR.20

Descriptive Statistics

1. Summarize, represent, and interpret data on two categorical and quantitative variables. MQR.DS.1

    21. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. M.QR.21

2. Summarize, represent, and interpret data on a single count or measurement variable. MQR.DS.2

    22. Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots). M.QR.22

    23. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets. M.QR.23

    24. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). M.QR.24

3. Perform statistical analysis. MQR.DS.3

    25. Create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features. M.QR.25

4. Communicate statistical information. MQR.DS.4

    26. Report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied. M.QR.26

    27. Communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language. M.QR.27

Probability

1. Analyze information using probability and counting. MQR.P.1
    28. Use the Fundamental Counting Principle, permutations and combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers. M.QR.28
    29. Determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables, and area models, to make decisions in problem situations. M.QR.29
2. Use probability to evaluate outcomes and manage uncertainty. MQR.P.2
    30. Use probabilities to make and justify decisions about risks in everyday life. M.QR.30
    31. Calculate expected value to analyze mathematical fairness, payoff, and rise. M.QR.31
3. Understand independence and conditional probability and use them to interpret data. MQR.P.3
    32. Describe events as subsets of a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events. M.QR.32
    33. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities. Use this characterization to determine if they are independent. M.QR.33
    34. Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. M.QR.34
    35. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. M.QR.35
    36. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. M.QR.36
4. Use the rules of probability to compute probabilities of compound events in a uniform probability model. MQR.P.4
    37. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. M.QR.37
    38. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model. M.QR.38
    39. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B) and interpret the answer in terms of the model. M.QR.39
    40. Use permutations and combinations to compute probabilities of compound events and solve problems. M.QR.40
5. Use probability to evaluate outcomes of decisions. MQR.P.5
    41. Use probabilities to make fair decisions. M.QR.41
    42. Analyze decisions and strategies using probability concepts. M.QR.42

Statistics

Descriptive Statistics

1. Summarize, represent, and interpret data on single count or measurement variable. S.DS.1
    1. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. M.PS.1
    2. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. M.PS.2
    3. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. M.PS.3
    4. Evaluate reports based on data. Write a function that describes a relationship between two quantities. M.PS.4
    5. Represent data with plots on the real number line (dots plots, histograms, and box plots). M.PS.5
    6. Use statistics appropriate to the shape of the data distributions to compare center and spread of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). M.PS.6
    7. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. M.PS.7
    8. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. M.PS.8

Probability

2. Understand independence and conditional probability and use them to interpret data. S.DS.2

    9. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). M.PS.9

    10. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent. M.PS.10

    11. Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. M.PS.11

    12. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. M.PS.12

    13. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. M.PS.13

    14. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. M.PS.14

    15. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model. M.PS.15

    16. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B) and interpret the answer in terms of the model. M.PS.16

    17. Use permutations and combinations to compute probabilities of compound events and solve problems. M.PS.17

Probability Distributions

3. Calculate expected values and use them to solve problems. S.DS.3

    18. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. M.PS.18

    19. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. M.PS.19

    20. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated to find the expected value. M.PS.20

    21. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically to find the expected value. M.PS.21

    22. Weight the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values (e.g., find the expected payoff for a game of chance). M.PS.22

    23. Evaluate and compare strategies on the basis of expected values. M.PS.23

    24. Analyze decisions and strategies using probability concepts. M.PS.24

Correlation and Regression

4. Interpret linear models. S.DS.4

    25. Represent data on two quantitative variables on a scatter plot and describe how the variables are related. M.PS.25

        a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. M.PS.25.A

        b. Informally assess the fit of a function by plotting and analyzing residuals. Focus should be on situations for which linear models are appropriate. M.PS.25.B

        c. Fit a linear function for scatter plots that suggest a linear association. M.PS.25.C

    26. Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit. M.PS.26

    27. Distinguish between correlation and causation. M.PS.27

Confidence Intervals

5. Determine and interpret confidence intervals. S.DS.5
    28. Find the point estimate and margin of error in a given scenario. M.PS.28
    29. Construct and interpret confidence intervals for the population mean. M.PS.29
    30. Determine minimum sample size requirements when estimating mean, μ (population proportion). M.PS.30
    31. Interpret the t-distribution and use t-distribution table in real-world scenarios. M.PS.31
    32. Construct confidence intervals when the sample size, n, is less than 30, population is normally distributed, and standard deviation, σ, is unknown. M.PS.32
    33. Interpret the chi-square distribution and use chi-square distribution table. Use the chi-square distribution to construct a confidence interval for the variance and standard deviation. M.PS.33

Hypothesis Testing with One Variable

6. Use hypothesis testing in making and interpreting decisions. S.DS.6
    34. Interpret a hypothesis test; state a null hypothesis and an alternative hypothesis. M.PS.34
    35. Identify Type I and Type II errors and interpret the level of significance. M.PS.35
    36. Use one-tailed and two-tailed statistical tests to find p-value. M.PS.36
    37. Make and interpret decisions on comparing two hypotheses based on results of a statistical test. Write a claim for a hypothesis test. M.PS.37
    38. Find probability values and test for mean. Use in a z-test. M.PS.38
    39. Find critical values and rejection regions in a normal distribution. Use rejection regions for a z-test. M.PS.39
    40. Find critical values in a t-distribution and use the t-test to test a mean. M.PS.40
    41. Use the z-test to test a population proportion, p. M.PS.41
    42. Find critical values for chi squared test. Use the chi squared test to test a variance or a standard deviation. M.PS.42

Statistical Inference

7. Determine and use correlation. S.DS.7
    43. Find a correlation coefficient. M.PS.43
    44. Test a population correlation coefficient using a table. M.PS.44
    45. Perform a hypothesis test for a population correlation coefficient. M.PS.45
    46. Distinguish between correlation and causation. M.PS.46
8. Use linear regression to predict and interpret. S.DS.8
    47. Find the equation of a regression line; predict y-values using a regression line. M.PS.47
    48. Interpret the types of variation about a regression line. M.PS.48
    49. Find and interpret the coefficient of determination. M.PS.49
    50. Find and interpret the standard error of estimate for a regression line; construct and interpret a prediction interval for y. M.PS.50
    51. Use technology to find a multiple regression equation, the standard error of estimate, and the coefficient of determination. M.PS.51
9. Use statistical tests to determine a relationship. S.DS.9
    52. Use a contingency table to find expected frequencies. M.PS.52
    53. Use the chi-squared distribution to test whether a frequency distribution fits a claimed distribution and to test whether two variables are independent. M.PS.53
    54. Interpret the F-distribution and use an F-table to find critical values. M.PS.54
    55. Perform a two-sample F-test to compare two variances. M.PS.55
    56. Perform a two-sample F-test to compare two variances. Interpret the F-distribution and use an F-table to find critical values. M.PS.56
    57. Use one-way analysis of variance to test claims involving three or more means. Introduce two-way analysis of variance. M.PS.57

Introduction to Mathematical Applications

Number and Quantity

1. Mathematics as a language. I.NQ.1
    1. Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments and analyzing the soundness of mathematical arguments of others. M.IMA.1
    2. Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems. M.IMA.2
    3. Use units to understand problems and to guide the solutions of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. M.IMA.3
2. Mathematics and Measurement I.NQ.2
    4. Select and correctly use an appropriate tool (e.g., tape measure, ruler, compass, level, micrometer, scale, protractor, thermometer, speedometer, odometer, pressure gauge, measuring squares, multimeter) to measure and/or calculate lengths, distances, directions, masses, temperatures, rates of change (e.g., slope, speed), areas, volumes, voltages, currents, and resistances. M.IMA.4
    5. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. M.IMA.5
    6. Solve real-world problems requiring conversion of units using dimensional analysis for measurements in English and metric systems. Solve problems involving multiple units of measurement (e.g., converting between currencies, calculating dosages of medicine, trip planning from miles to kilometers). M.IMA.6
    7. Distinguish between proportional and non-proportional situations, apply proportional reasoning when appropriate, solve for an unknown quantity in proportional situations; apply scale factors to perform indirect measurements using maps, blueprints, concentrations, dosages, and densities. M.IMA.7
3. The Real Number System. I.NQ.3
    8. Perform operations and convert quantities between fractions, decimals, and percents using positive and negative numbers, fractions, absolute value, decimals, percentages, and scientific notation (e.g., given the cost of a project, determine what percentage of the budget were salaries; percent of increase/decrease). M.IMA.8
    9. Solve real-world problems in a variety of contexts by representing quantities in equivalent forms (fractions, decimals, and percentages) to investigate and describe quantitative relationships. Compare the size of numbers in different forms arising in authentic real-world contexts, such as growth expressed as a fraction versus as a percentage. Interpret the meaning of numbers in different forms, such as scientific notation and the meaning of a fraction or percentage greater than 100 and its validity in a given context. Recognize incorrect or deceptive uses of fractions, decimals, or percentages. M.IMA.9

Algebra – Seeing Structure in Expressions

1. Understand the connections between proportional relationships, lines, and linear equations. `I.IS.1`

    10. Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed (e.g., labor cost per time, material cost per job). `M.IMA.10`

    11. Solve application problems using direct and inverse variation equations (e.g., determine the mechanical advantage of gears, Ohm's Law). `M.IMA.11`

2. Create equations that describe numbers or relationships. `I.IS.2`

    12. Analyze real-world problem situations and use variables to construct and solve equations involving one or more unknown or variable quantities to answer questions about the situations, such as creating spreadsheet formulas to calculate prices based on percentage mark-up or solving formulas for specified values. Demonstrate understanding of the meaning of a solution. Identify when there is insufficient information given to solve a problem. `M.IMA.12`

    13. Analyze real-world problem situations and use variables to construct and solve equations and inequalities in one variable, representing linear, exponential, and simple rational functions (e.g., using spreadsheet functions, determine sale price of items). `M.IMA.13`

    14. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales (e.g., profit vs. number of units, cost vs. number of units, resistance vs. current). `M.IMA.14`

    15. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations (e.g., rearrange Ohm's law V = IR to highlight resistance R). `M.IMA.15`

3. Solve systems of equations. `I.IS.3`

    16. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables (e.g., childcare facility – sq. footage to number of children; solving electrical current in a circuit with multiple paths, break-even point). `M.IMA.16`

Functions – Interpreting Functions

1. Understand the concept of a function and use function notation. I.F.1

    17. Use multiple representations of functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically. M.IMA.17

2. Analyze functions using different representations. I.F.2

    18. Interpret the parameters in a linear function in terms of a context. M.IMA.18

    19. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. M.IMA.19

    20. Describe qualitatively the functional relationship between two quantities by analyzing a graph. M.IMA.20

3. Build a function that models a relationship between two quantities. I.F.3

    21. Represent application problems as linear equations. Write a function that describes a relationship between two quantities (e.g., level of education versus pay; rate of speed versus fuel consumption; caloric intake versus expenditure). M.IMA.21

    22. Recognize that the graph of a linear or exponential equation in two variables is the set of all its solutions plotted in the coordinate plane. M.IMA.22

Geometry/Trigonometry

1. Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations. I.GT.1

    23. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects (e.g., three-view drawings and blueprints). M.IMA.23

    24. Use two- and three-dimensional shapes and circles, their measures, and their properties to describe objects. M.IMA.24

        a. Apply concepts of density based on area and volume in modeling situations. M.IMA.24.A

        b. Apply geometric methods to solve design problems to satisfy given constraints (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios). M.IMA.24.B

2. Use geometric theorems and formulas to solve problems. I.GT.2

    25. Explore theorems about triangles to solve real-world application problems. M.IMA.25

    26. Understand and apply the Pythagorean Theorem for solving real-world problems (e.g., checking accuracy on gate construction, conduit bending, roof pitch). M.IMA.26

    27. Solve application problems by calculating area and surface area for two-dimensional objects (e.g., calculate the cost of installing flooring in a building and painting the interior and exterior of a building based on square footage). M.IMA.27

    28. Solve application problems by calculating volume for three-dimensional objects using formulas for cylinders, pyramids, prisms, cones, and spheres (e.g., compute amount of cement needed for a sidewalk, amount of water in a fire hose, amount of air in ductwork). M.IMA.28

    29. Solve application problems by calculating circumference, area, radius, diameter, area of sector, arc length of a circle with appropriate unit labels (e.g., develop a circular watering system). M.IMA.29

Modeling

1. Concrete geometric representation (physical modeling) `I.M.1`
    30. Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications. `M.IMA.30`
    31. Gather data, conduct investigations, and apply mathematical concepts and models to solve problems (e.g., designing and building a house or a car). `M.IMA.31`
2. Summarize, represent, and interpret data on two quantitative variables. `I.M.2`
    32. Collect numerical bivariate data; represent data on two quantitative variables on a scatter plot; determine whether or not a relationship exists; if so, describe how the variables are related and select a function to model the data, justify the selection and use the model to make predictions (e.g., cost of the materials for a construction project, cost of the labor for a project, cost and value of a vehicle based on depreciation). `M.IMA.32`
    33. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. Interpret the rate of change and the constant term of a linear model in the context of the data. `M.IMA.33`
    34. Identify positive and negative correlations (e.g., vehicle depreciation). Use technology to compute and interpret the correlation coefficient of a linear fit. `M.IMA.34`

Statistics and Probability: Interpreting Categorical & Quantitative Data

1. Summarize, represent, and interpret data on a single count or measurement variable. `I.SP.1`
    35. Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots). `M.IMA.35`
    36. Analyze and interpret tables, charts, and graphs (e.g., interpret a body mass index (BMI) chart). `M.IMA.36`
    37. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets. `M.IMA.37`
    38. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). `M.IMA.38`
    39. Distinguish between correlation and causation. `M.IMA.39`

Finance Mathematics

1. Understanding financial models. `I.FM.1`

    40. Determine, represent, and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., business loans, auto, mortgage, and/or credit card). `M.IMA.40`

    41. Determine, represent, and analyze mathematical models for investments involving simple and compound interest with and without additional deposits (e.g., savings accounts, bonds, and/or certificates of deposit). `M.IMA.41`

2. Personal use of finance. `I.FM.2`

    42. Research, develop, and analyze personal budgets based on given parameters (e.g., fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission, career choice, geographic region, retirement, and/or investment planning). `M.IMA.42`

    43. Research and analyze taxes including payroll, sales, personal property, real estate, and income tax returns. `M.IMA.43`