Ohio Mathematics

High School — Functions

Adopted 2017

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.** MP.1
2. **Reason abstractly and quantitatively.** MP.2
3. **Construct viable arguments and critique the reasoning of others.** MP.3
4. **Model with mathematics.** MP.4
5. **Use appropriate tools strategically.** MP.5
6. **Attend to precision.** MP.6
7. **Look for and make use of structure.** MP.7
8. **Look for and express regularity in repeated reasoning.** MP.8

Interpreting Functions

A. **Understand the concept of a function, and use function notation.** HSF-IF.A

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.1
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.2
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.IF.3

B. Interpret functions that arise in applications in terms of the context. HSF-IF.B

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. F.IF.4
    a. Focus on linear and exponential functions. F.IF.4.A
    b. Focus on linear, quadratic, and exponential functions. F.IF.4.B
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.5
    a. Focus on linear and exponential functions. F.IF.5.A
    b. Focus on linear, quadratic, and exponential functions. F.IF.5.B
    c. Emphasize the selection of a type of function for a model based on behavior of data and context. F.IF.5.C
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.IF.6

C. **Analyze functions using different representations.** HSF-IF.C

7. Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. F.IF.7

   a. Graph linear functions and indicate intercepts. F.IF.7.A

   b. Graph quadratic functions and indicate intercepts, maxima, and minima. F.IF.7.B

   c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.7.C

   d. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. F.IF.7.D

   e. Graph simple exponential functions, indicating intercepts and end behavior. F.IF.7.E

   f. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.7.F

   g. Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior. (+)F.IF.7.G

   h. Graph logarithmic functions, indicating intercepts and end behavior. (+)F.IF.7.H

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.8

   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F.IF.8.A

      i. Focus on completing the square to quadratic functions with the leading coefficient of 1. F.IF.8.A.I

   b. Use the properties of exponents to interpret expressions for exponential functions. F.IF.8.B

      i. Focus on exponential functions evaluated at integer inputs. F.IF.8.B.I

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.9

   a. Focus on linear and exponential functions. F.IF.9.A

   b. Focus on linear, quadratic, and exponential functions. F.IF.9.B

Building Functions

A. Build a function that models a relationship between two quantities. HSF-BF.A

1. Write a function that describes a relationship between two quantities. F.BF.1
   a. Determine an explicit expression, a recursive process, or steps for calculation from context. F.BF.1.A
      i. Focus on linear and exponential functions. F.BF.1.A.I
      ii. Focus on situations that exhibit quadratic or exponential relationships. F.BF.1.A.II
   b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. F.BF.1.B
   c. Compose functions. (+)F.BF.1.C
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.BF.2

B. Build new functions from existing functions. HSF-BF.B

3. Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.3
   a. Focus on transformations of graphs of quadratic functions, except for f(kx); F.BF.3.A
4. Find inverse functions. F.BF.4
   a. Informally determine the input of a function when the output is known. F.BF.4.A
   b. Read values of an inverse function from a graph or a table, given that the function has an inverse. (+)F.BF.4.B
   c. Verify by composition that one function is the inverse of another. (+)F.BF.4.C
   d. Find the inverse of a function algebraically, given that the function has an inverse. (+)F.BF.4.D
   e. Produce an invertible function from a non-invertible function by restricting the domain. (+)F.BF.4.E
5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. (+)F.BF.5

Linear, Quadratic, And Exponential Models

A. Construct and compare linear, quadratic, and exponential models, and solve problems. HSF-LE.A

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.1
   a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. F.LE.1.A
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. F.LE.1.B
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.1.C
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.2
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. F.LE.3
4. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. F.LE.4

B. Interpret expressions for functions in terms of the situation they model. HSF-LE.B

5. Interpret the parameters in a linear or exponential function in terms of a context. F.LE.5

Trigonometric Functions

A. Extend the domain of trigonometric functions using the unit circle. HSF-TF.A

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.1
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counter-clockwise around the unit circle. F.TF.2
3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. (+)F.TF.3
4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. (+)F.TF.4

B. Model periodic phenomena with trigonometric functions. HSF-TF.B

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F.TF.5
6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. (+)F.TF.6
7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. (+)F.T.7

C. Prove and apply trigonometric identities. HSF-TF.C

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. F.TF.8
9. Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems. (+)F.TF.9