Ohio Mathematics - Extended Learning Standards

Grade 4

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.1

Complexity a

a Solve a multiplicative comparison using a visual and/or physical representation (limited to multiples of 2s, 3s, 4s, 5s, 6s, and 10s). 4.OA.1A

Complexity b

b Solve a multiplicative comparison problem using linear or other physical representations (limit to whole number factors of 2s, 4s, 5s, and 10s). 4.OA.1B

Complexity c

c Solve a multiplicative comparison problem using linear models or other physical representations (limit to whole number factors of 2s and 5s). 4.OA.1C

Learning Progression

- Identify 2 and 5 blocks. 4.OA.1.LP.A
- Identify groups of blocks 2s and 5s. 4.OA.1.LP.B
- Build groups of blocks into rows and columns (arrays). 4.OA.1.LP.C
- Count the number of blocks in a given array. 4.OA.1.LP.D
- Build an array and count the number of blocks. 4.OA.1.LP.E
- Skip count by 2s and 5s. 4.OA.1.LP.F
- Identify the number of blocks in each row and each column. 4.OA.1.LP.G
- Match an array to its factors. 4.OA.1.LP.H
- Listen to uses of multiplicative comparisons (times as much or times as many) in real-world contexts. (Ohio Learning Standards for Mathematics, pg. 96) 4.OA.1.LP.I
- Engagement Statements (demonstration of engaged in the topic) 4.OA.1.LP.J
- Interact with linear models or physical objects (blocks). 4.OA.1.LP.K

2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) 4.OA.2

Complexity a

a Solve for the unknown product or quotient when given a word problem involving a multiplicative comparison (limit to whole number factors to 10 x 10). 4.OA.2A

Complexity b

b Given an array, area model, or other physical model that best represents a multiplicative comparison in a word problem, solve for the unknown product or quotient (limit to whole number factors of the 2s, 4s, 5s, and 10s). 4.OA.2B

Complexity c

c Identify the array, area model, or other physical representation that best represents a multiplicative comparison in a word problem (limit to multiples of 2s and 5s). 4.OA.2C

Learning Progression

- Identify 2 and 5 blocks. 4.OA.2.LP.A
- Identify groups of blocks 2s and 5s. 4.OA.2.LP.B
- Build groups of blocks into rows and columns (arrays). 4.OA.2.LP.C
- Count the number of blocks in a given array. 4.OA.2.LP.D
- Build an array and count the number of blocks. 4.OA.2.LP.E
- Skip count by 2s and 5s. 4.OA.2.LP.F
- Identify the number of blocks in each row and each column. 4.OA.2.LP.G
- Match an array to its factors. 4.OA.2.LP.H
- Read multiplicative comparison statements (times as much or times as many) in real-world contexts. (Ohio Learning Standards for Mathematics, pg. 96) 4.OA.2.LP.I
- Solve 1-step number sentences involving multiplication or division. 4.OA.2.LP.J
- Engagement Statements (demonstration of engaged in the topic) 4.OA.2.LP.K
- Interact with linear models or physical objects (blocks). 4.OA.2.LP.L

3 Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 4.OA.3

Complexity a

a Determine the operation and correctly solve one-step word problems with remainders when given visual and/or physical representations (whole numbers within 1,000). 4.OA.3A

Complexity b

b Determine the operation(s) and correctly solve twostep word problems, without remainders, when given visual and/or physical representations (whole numbers; sums to 100). 4.OA.3B

Complexity c

c Solve a one-step word problem using a given visual and/ or physical model (whole numbers; sums to 30; factors of 1s, 2s, 5s, and 10s). 4.OA.3C

Learning Progression

- Identify a number sentence. 4.OA.3.LP.A
- Count the number of objects in an array. 4.OA.3.LP.B
- Recognize the symbols for addition (+), subtraction, (–), multiplication (×), division (÷), and equals (=). 4.OA.3.LP.C
- Read and interpret a traditional one-step number sentence (2 × 3 = □). 4.OA.3.LP.D
- Relate a picture or objects to a number sentence. 4.OA.3.LP.E
- Know that a symbol □ can represent a missing value. 4.OA.3.LP.F
- Count to 30. 4.OA.3.LP.G
- Count physical objects up to 30. 4.OA.3.LP.H
- Identify groups of blocks 2s, 5s, and 10s. 4.OA.3.LP.I
- Build groups of blocks into rows and columns (arrays). 4.OA.3.LP.J
- Count the number of blocks in a given array. 4.OA.3.LP.K
- Build an array and count the number of blocks. 4.OA.3.LP.L
- Identify the number of blocks in each row and each column. 4.OA.3.LP.M
- Match an array to its factors. 4.OA.3.LP.N
- Engagement Statements (demonstration of engaged in the topic) 4.OA.3.LP.O
- Interact with physical objects (blocks) or drawings representing addition, subtraction, or multiplication word problems. 4.OA.3.LP.P

Gain familiarity with factors and multiples.

4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. 4.OA.4

Complexity a

a Using a multiplication table or other tool, identify the factor pairs for whole numbers in the range of 1–50. 4.OA.4A

Complexity b

b Match and/ or sort factor pairs for numbers up to 50 with physical and/or visual representations (for example, 50 matches: 1×50, 50×1, 10×5, 5×10, 2×25, and 25×2; 37 matches 37×1 and 1×37). 4.OA.4B

Complexity c

c Match factor pairs for whole numbers up to 20 with physical and/or visual representations (for example, 12 matches: 1×12 and 12×1, 3×4 and 4×3, and 6×2 and 2×6). 4.OA.4C

Learning Progression

- Count the number of objects in an array. 4.OA.4.LP.A
- Relate a picture or objects to a number sentence. 4.OA.4.LP.B
- Recognize the symbols for multiplication (×) and equals (=). 4.OA.4.LP.C
- Build groups of blocks into rows and columns (arrays). 4.OA.4.LP.D
- Identify the number of blocks in each row, column, and in total. 4.OA.4.LP.E
- Create multiple arrays using the same number of blocks. 4.OA.4.LP.F
- Recognize the orientation of an array can change. 4.OA.4.LP.G
- Recognize factors in an array are the same when two arrays are represented with different orientations. 4.OA.4.LP.H
- Match an array to its factors. 4.OA.4.LP.I
- Write a number sentence that represents a given array. 4.OA.4.LP.J
- Write a number sentence in more than one way. 4.OA.4.LP.K
- Engagement Statements (demonstration of engaged in the topic) 4.OA.4.LP.L
- Interact with physical objects (blocks) or drawings representing addition, subtraction, or multiplication word problems. 4.OA.4.LP.M

Generate and analyze patterns.

5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. 4.0A.5

Complexity a

a Given a rule for a pattern and its visual and/or physical representation, extend the pattern or identify or exclude objects or numbers that don’t fit the rule of the pattern from physical and/or visual representations. 4.0A.5A

Complexity b

b Extend a shape or number pattern up to five terms given physical and/or visual representations. 4.0A.5B

Complexity c

c Extend a shape pattern two terms using a visual or physical representation (manipulatives). 4.0A.5C

Learning Progression

- Create a two-dimensional shape using triangles, rectangles, or squares when given physical objects such as pattern blocks. 4.0A.5.LP.A
- Create a two-dimensional shape using triangles, rectangles, or squares. 4.0A.5.LP.B
- Compose a larger two-dimensional shape from an original two-dimensional using triangles, rectangles, or squares when given physical objects such as pattern blocks. 4.0A.5.LP.C
- Match a given pattern composed triangles, rectangles, or squares. 4.0A.5.LP.D
- Identify the differences between an initial shape and its grown shape. ° Larger or smaller ° Colors of shapes ° Orientations ° Specific additions of shapes 4.0A.5.LP.E
- Engagement Statements (demonstration of engaged in the topic) 4.0A.5.LP.F
- Interact with physical objects (blocks) or drawings. 4.0A.5.LP.G

Numbers and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.

1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division. 4.NBT.1

Complexity a

a Decompose multi-digit whole numbers by their place values and expanded form up to 100,000 with physical and/or visual representations (for example, 457: 4 hundreds, 5 tens, 7 ones; four hundred fiftyseven; 400 + 50 + 7). 4.NBT.1A

Complexity b

b1 Given a whole number within the range of 1-999, decompose into place values of ones, tens and hundreds using physical and/or visual representations. AND 4.NBT.1B1

b2 Given a whole number within the range of 1–999, identify the expanded form using physical and/or visual representations. 4.NBT.1B2

Complexity c

c Given a whole number within the range of 1–99, identify the value in the ones place and/ or the tens place using a place value chart or other visual/physical representation. 4.NBT.1C

Learning Progression

- Count to 100 by ones and by tens. 4.NBT.1.LP.A
- Recognize the numerals from 1 to 100. 4.NBT.1.LP.B
- Represent numbers from 1 to 100 using physical objects. 4.NBT.1.LP.C
- Know the word names for the numbers 1-100. 4.NBT.1.LP.D
- Write numerals from 0 to 100. 4.NBT.1.LP.E
- Explore place value tools. ° Base-10 blocks ° Place value chart ° 100's chart ° Cuisenaire rods ° Unifix cubes 4.NBT.1.LP.F
- Distribute objects into groups of tens and ones. ° Compose and decompose numbers from 11 to 19 into a group of ten ones and some further ones by using objects. ° Understand that these numbers are composed of a group of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. 4.NBT.1.LP.G
- Record the number of tens and ones in a group of objects or drawings. 4.NBT.1.LP.H
- Understand that the two digits of a two-digit number represent amounts of tens and ones. 4.NBT.1.LP.I
- Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a "ten;" 4.NBT.1.LP.J
- Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). 4.NBT.1.LP.K

- Identify the location of the ones and tens on a place value chart. 4.NBT.1.LP.L
- Engagement Statements (demonstration of engaged in the topic) 4.NBT.1.LP.M
- Interact with physical objects (blocks) or drawings. 4.NBT.1.LP.N

2 Read and write multidigit whole numbers using standard form, word form, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. 4.NBT.2

Complexity a

a1 Use place value knowledge to compare 2 numbers using >, =, and < symbols along with physical and/or visual representations (whole numbers 1–10,000). 4.NBT.2A1

a2 Read and write numbers up to 10,000 in standard and expanded form. 4.NBT.2A2

Complexity b

b1 Use place value knowledge to compare 2 numbers using >, <, = symbols along with physical and/or visual representations (whole numbers 1–1000). 4.NBT.2B1

b2 Given a number in standard form or word form, write the number in expanded form. For example, 206 = 200 + 6 (whole numbers 1–1000). 4.NBT.2B2

Complexity c

c Match the word form or standard form of two-digit whole numbers with physical and/or visual representations of objects and place values. For example, “25” or the word “twentyfive” is matched to a set of 25 objects and/or 2 tens and 5 ones cubes (whole numbers to 99). 4.NBT.2C

Learning Progression

- Recognize the numerals from 1 to 100. 4.NBT.2.LP.A
- Represent numbers from 1 to 100 using physical objects. 4.NBT.2.LP.B
- Know the word names for the numbers 1-100. 4.NBT.2.LP.C
- Write numerals from 0 to 100. 4.NBT.2.LP.D
- Explore place value tools. ° Base-10 blocks ° Place value chart ° 100’s chart ° Cuisenaire rods ° Unifix cubes 4.NBT.2.LP.E
- Distribute objects into groups of tens and ones. 4.NBT.2.LP.F
- Record the number of tens and ones in a group of objects or drawings. 4.NBT.2.LP.G
- Understand that the two digits of a two-digit number represent amounts of tens and ones. 4.NBT.2.LP.H

- Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten;” 4.NBT.2.LP.I
- Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). ° Identify the location of the ones and tens on a place value chart. 4.NBT.2.LP.J
- Recognize the standard form of a number when given the word name. 4.NBT.2.LP.K
- Engagement Statements (demonstration of engaged in the topic) 4.NBT.2.LP.L
- Interact with physical objects (blocks) or drawings. 4.NBT.2.LP.M

3 Use place value understanding to round multidigit whole numbers to any place through 1,000,000. 4.NBT.3

Complexity a

a Round whole numbers to any place using physical and/ or linear visual representations (whole numbers to 10,000, number lines). 4.NBT.3A

Complexity b

b Round whole numbers to the nearest 10 or 100 using physical and/or linear visual representations (whole numbers to 1,000, number lines). 4.NBT.3B

Complexity c

c Round two-digit whole numbers to the nearest 10 using a physical and/ or linear visual representation (whole numbers to 99, number lines). 4.NBT.3C

Learning Progression

- Know what a number line is. 4.NBT.3.LP.A
- Know the order of the numbers from 0 to 99. 4.NBT.3.LP.B
- Identify a whole number on a number line marked with whole numbers up to 99. 4.NBT.3.LP.C
- Identify 0 on a number line. 4.NBT.3.LP.D
- Identify a missing whole number value on a number line marked with whole number up to 99. 4.NBT.3.LP.E
- Compare distances of objects using a number line. 4.NBT.3.LP.F
- Understand that 2 is the distance from 0 to 2 and 3 is the distance from 0 to 3 using standard units for all lengths from 1 to 99. 4.NBT.3.LP.G
- Determine which tens a number comes between using a vertical or horizontal number line. 4.NBT.3.LP.H
- Engagement Statements (demonstration of engaged in the topic) 4.NBT.3.LP.I
- Interact with physical objects (blocks) or drawings (may include 100s chart) representing whole numbers up to 99. 4.NBT.3.LP.J

Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.

4 Fluently add and subtract multi-digit whole numbers using a standard algorithm. 4.NBT.4

Complexity a

a Add and subtract (with regrouping) 3-digit whole numbers using place value strategies and/or physical or visual representations (sums within 10,000). 4.NBT.4A

Complexity b

b Add and subtract up to two 3-digit whole numbers using place value strategies and/or physical or visual representations (including: adding two 2-digit whole numbers whose sums are less than 100 and may require regrouping; and adding two 3-digit numbers without regrouping whose sums are less than 1000; subtraction of two 2-digit or two 3-digit numbers without regrouping). 4.NBT.4B

Complexity c

c Add and subtract whole numbers using place value strategies and/or physical or visual representations. (Including sums of three one-digit whole numbers within 30, sums of 1-digit and 2-digit whole numbers with regrouping allowed in ones, and sums of two 2-digit whole numbers whose sums are within 100 without regrouping; subtraction of up to two 2-digit numbers without regrouping whose sums are within 100). 4.NBT.4C

Learning Progression

- Represent a number with a set of physical objects or a drawing. 4.NBT.4.LP.A
- Understand addition is the combining of two (or more) sets of objects. 4.NBT.4.LP.B
- Understand subtraction is taking away of one amount of objects from another. 4.NBT.4.LP.C
- Understand that addition and subtraction are opposites. 4.NBT.4.LP.D
- Know the symbols for addition (+), subtraction, (–), and equals (=). 4.NBT.4.LP.E
- Add and subtract within 20 using strategies. Strategies may include: 4.NBT.4.LP.F
- Counting on 4.NBT.4.LP.G
- Making ten (8 + 6 = 8 + 2 + 4 = 10 + 4 = 14) 4.NBT.4.LP.H
- Decomposing a number leading to a ten (13 − 4 = 13 − 3 − 1 = 10 − 1 = 9) 4.NBT.4.LP.I
- Using the relationship between addition and subtraction; knowing that 4.NBT.4.LP.J
- 8 + 4 = 12, one knows 12 − 8 = 4 and 4.NBT.4.LP.K

- Creating equivalent but easier or known sums (adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). 4.NBT.4.LP.L
- Engagement Statements (demonstration of engaged in the topic) 4.NBT.4.LP.M
- Interact with physical objects (blocks) or drawings (may include 100s chart) representing whole numbers within 30. 4.NBT.4.LP.N

5 Multiply a whole number of up to four digits by a 1-digit whole number, and multiply two 2-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.5

Complexity a

a Multiply a 2-digit whole number by a 2- digit whole number, using strategies based on place value and the properties of operations with arrays, area models, or other visual representations (products within 10,000). 4.NBT.5A

Complexity b

b Multiply a 3-digit whole number by a 1-digit whole number, using strategies based on place value and the properties of operations along with arrays, area models, or other physical representations (products within 1,000). 4.NBT.5B

Complexity c

c Multiply a multiple of 10 by a 1-digit whole number using arrays, area models, or other physical representations. 4.NBT.5C

Learning Progression

- Create multiple groups of 10 using objects. 4.NBT.5.LP.A
- Repeatedly add groups of 10 using physical objects. 4.NBT.5.LP.B
- Count to 100 by 10s. 4.NBT.5.LP.C
- Relate counting to addition by counting on 10 to add 10. 4.NBT.5.LP.D
- Represent a number with a set of physical objects or a drawing. 4.NBT.5.LP.E
- Record the number of tens and 0 ones in a group of objects or drawings for multiples of 10. 4.NBT.5.LP.F
- Understand that the two digits of a two-digit number represent amounts of tens and ones. 4.NBT.5.LP.G
- Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten”. 4.NBT.5.LP.H
- Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). 4.NBT.5.LP.I
- Engagement Statements (demonstration of engaged in the topic) 4.NBT.5.LP.J
- Interact with physical objects (blocks) or drawings (100s chart or multiplication chart). 4.NBT.5.LP.K

6 Find whole-number quotients and remainders with up to 4-digit dividends and 1-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.6

Complexity a

a Divide a 3-digit whole number by a 1-digit whole number using strategies based on place value, relationship between multiplication and division and the properties of operations using arrays, area models or other physical/visual representations (whole numbers answers with no remainders). 4.NBT.6A

Complexity b

b Divide multiples of 10 up to 90 by a 1-digit whole number using strategies based on place value, the relationship between multiplication and division, and the properties of operations using arrays, area models, or other physical/visual representations (whole numbers to 90 with no remainders). 4.NBT.6B

Complexity c

c Determine whether a number is divisible by 2, 5, or 10 for numbers up to 50 using physical and/or visual representations (whole numbers to 100). 4.NBT.6C

Learning Progression

- Identify groups of blocks 2s, 5s, and 10s. 4.NBT.6.LP.A
- Share up to 50 objects equally between 2, 5, and 10 people (without remainders). 4.NBT.6.LP.B
- Engagement Statements (demonstration of engaged in the topic) 4.NBT.6.LP.C
- Interact with physical objects (blocks). 4.NBT.6.LP.D

Numbers and Operations – Fractions

Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.1

Complexity a

a Write or model equivalent fractions for denominators 2, 3, 4, 5, 6, 8, 10 when given rectangular fraction models or other physical/ visual models. (fraction strips, number lines, area models). 4.NF.1A

Complexity b

b Identify equivalent fractions for denominators 2, 3, 4, 5, 6, 8, 10 when given rectangular fraction models or other physical visual models, for example, matching model of 1/2 and 2/4. 4.NF.1B

Complexity c

c Match fractions with their fraction model or other physical/ visual models (limit to 1/3, 2/3, ¼, ½, and 3/4). 4.NF.1C

Learning Progression

- Identify the same sized whole partitioned into 2, 3, and 4 equal shares. 4.NF.1.LP.A
- Describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. 4.NF.1.LP.B
- Describe the whole as two halves, three thirds, or four fourths. 4.NF.1.LP.C
- Recognize the value of a whole fractional shaded part. 4.NF.1.LP.D
- Identify ¼, 1/3, and ½ when given a fraction model. 4.NF.1.LP.E
- Engagement Statements (demonstration of engaged in the topic) 4.NF.1.LP.F
- Interact with area (rectangles) and length (number lines) fraction models. 4.NF.1.LP.G

2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. 4.NF.2

Complexity a

a Compare two fractions using models and >, =, and < symbols (limit denominators to 2, 3, 4, 5, 6, 8, 10). 4.NF.2A

Complexity b

b Using a given model, compare two fractions to identify which is “greater than”, “less than”, or “equal to” (limit fractions to unit fractions with denominators of 2, 3, 4, 5, 6, 8, 10). 4.NF.2B

Complexity c

c Determine which fraction is larger or smaller given pairs of fractions and their models (limit to 1/3, 2/3, ¼, ½, and 3/4). 4.NF.2C

Learning Progression

- Directly compare two objects with a measurable attribute in common to see which object has “more of” or “less of” the attribute, and describe the difference. 4.NF.2.LP.A
- Directly compare the heights of two children, and describe one child as taller/shorter. 4.NF.2.LP.B
- Interact with physical objects and length models that represent fractional values. 4.NF.2.LP.C
- Visually compare fractional values using physical objects. 4.NF.2.LP.D
- Engagement Statements (demonstration of engaged in the topic) 4.NF.2.LP.E
- Interact with area (rectangles) and length (number lines) fraction models. 4.NF.2.LP.F

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified.)

3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. 4.NF.3

Complexity a

a Using physical and/or visual representation and fractions with denominators of 3, 4, 5, 6, 8, 10 and 100). a. Add and subtract fractions with like denominators. b. Decompose a mixed number into sums of fractions, for example, 1¾ = 4/4 + ¼ + ¼ + ¼. c. Add and subtract fractions with like denominators for example, 1 1/3+ 1 1/3= 2 2/3 or 1 2/3 - 1/3 = 1 1/3 (including mixed numbers). d. Solve one-step real-world problems involving addition or subtraction of fractions with like denominators (referring to the same whole). 4.NF.3A

Complexity b

b Using physical models or visual representations of fractions with denominators of 2, 3, 4, 5, 6, 8 and 10: a. Add and subtract fractions with like denominators. b. Decompose a mixed number into sums of unit fractions, for example, 1¾ = ¼ + ¼ + ¼ + ¼ + ¼ + ¼ + ¼. c. Add and subtract a mixed number with a fraction of the same denominator. d. Solve real-world problems involving addition or subtraction of fractions with like denominators (referring to the same whole). 4.NF.3B

Complexity c

c Using physical models or visual representations and fractions with denominators of 2, 3 and 4 a. Add fractions with like denominators. b. Decompose a fraction into sums of unit fractions, for example, ¾ = ¼ + ¼ + ¼ . c. Know that one whole (partitioned into equal-sized parts) equals the sum of all its equal parts. For example, 1 whole = 4 fourths (4/4) or 3 thirds (3/3). d. Solve onestep real-world problems involving addition of unit fractions (referring to the same whole). 4.NF.3C

Learning Progression

- Identify the same sized whole partitioned into 2, 3, and 4 equal shares. 4.NF.3.LP.A
- Describe the whole as two halves, three thirds, or four fourths. 4.NF.3.LP.B
- Recognize the value of a whole fractional shaded part. 4.NF.3.LP.C

- Identify ¼, 1/3, and ½ when given a fraction model. 4.NF.3.LP.D
- Partition rectangles into two, three, or four equal shares. 4.NF.3.LP.E
- Write a number sentence representing a whole partitioned into two, three, or four equal shares (4/4 = ¼ + ¼ + ¼ + ¼). 4.NF.3.LP.F
- Describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. 4.NF.3.LP.G
- Describe the whole as two halves, three thirds, or four fourths in real-world contexts. 4.NF.3.LP.H
- Understand a fraction 1/b is the smallest fractional part of a whole. 4.NF.3.LP.I
- Count by unit fractions up to a whole. (¼, 1/3, and ½) 4.NF.3.LP.J
- Use fraction models to combine equal sized shares with like denominators. 4.NF.3.LP.K
- Understand you can combine equal sized shares of fractional parts of the same whole like you can combine whole numbers. 4.NF.3.LP.L
- Engagement Statements (demonstration of engaged in the topic) 4.NF.3.LP.M
- Interact with area (rectangles) and length (number lines) fraction models. 4.NF.3.LP.N

4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4) or 5/4 = (1/4) + (1/4) + (1/4) + (1/4) + (1/4). b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.) c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? 4.NF.4

Complexity a

a Solve real-world problems involving multiplying a fraction by a whole number up to 10 using visual fraction models. For example, three friends each ate 2/8 of a pizza. How much pizza did they eat? (limit to fractions with denominators of 2, 3, 4, 6, and 8; no mixed numbers). 4.NF.4A

Complexity b

b Identify equivalent number sentences for fractions expressed as sums of unit fractions and products of a one-digit whole number multiplied by the same unit fraction, for example, ¾ = ¼ + ¼ + ¼ matches 3 × ¼ (limit to fractions with denominators of 2, 3, 4, 5, 6, 8 and 10; physical or visual fraction models may be used). 4.NF.4B

Complexity c

c Match a whole number with its given equivalent fraction using physical and/or visual representations for mathematical and word problems. (For example, 4=4/1). 4.NF.4C

Learning Progression

- Describe the whole as two halves, three thirds, or four fourths. 4.NF.4.LP.A
- Recognize the value of a whole fractional shaded part. 4.NF.4.LP.B
- Identify ¼, 1/3, and ½ when given a fraction model. 4.NF.4.LP.C
- Partition rectangles into two, three, or four equal shares. 4.NF.4.LP.D
- Write a number sentence representing a whole partitioned into two, three, or four equal shares (4/4 = ¼ + ¼ + ¼ + ¼). 4.NF.4.LP.E
- Describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. 4.NF.4.LP.F
- Describe the whole as two halves, three thirds, or four fourths in real-world contexts. 4.NF.4.LP.G
- Engagement Statements (demonstration of engaged in the topic) 4.NF.4.LP.H
- Interact with area (rectangles) and length (number lines) fraction models. 4.NF.4.LP.I

5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade. 4.NF.5

Complexity a

a Identify a fraction with a denominator of 10 as an equivalent fraction with denominator 100. For example, 30/10 = 3/10). 4.NF.5A

Complexity b

b Match a fraction with a denominator of 10 with its equivalent physical or visual model. 4.NF.5B

Complexity c

c Match a fraction with a denominator of 100 with its equivalent physical or visual model. 4.NF.5C

Learning Progression

- Understand content from 4.NF.1-4 prior to beginning instruction on 4.NF.5. 4.NF.5.LP.A
- Explore place values using place value models. (Base-10 blocks, Cuisenaire rods, pennies, dimes, etc.) 4.NF.5.LP.B
- Describe the whole as hundredths. 4.NF.5.LP.C

- Recognize the value of a fraction in hundredths using a model. 4.NF.5.LP.D
- Identify hundredths when given a fraction model. 4.NF.5.LP.E
- Partition rectangles into hundredths of equal shares. 4.NF.5.LP.F
- Describe the shares in terms of hundredths. 4.NF.5.LP.G
- Describe the shares in terms of hundredths in real-world contexts. 4.NF.5.LP.H
- Engagement Statements (demonstration of engaged in the topic) 4.NF.5.LP.I
- Interact with place value models. (Base-10 blocks, Cuisenaire rods, pennies, dimes, etc.) 4.NF.5.LP.J

6 Use decimal notation for fractions with denominators of 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. 4.NF.6

Complexity a

a Rewrite a fraction with a denominator of 100 as a decimal using place value visual and/or physical representations. For example, rewrite 62/100 as 0.62. 4.NF.6A

Complexity b

b Rewrite a fraction with a denominator of 10 as a decimal. For example, rewrite 2/10 as 0.2 using place value, physical and/or visual representations. 4.NF.6B

Complexity c

c Match a collection of pennies or dimes to the visual model of the decimal. AND Select the decimal that represents a visual and/or physical model for a collection of pennies or dimes. 4.NF.6C

Learning Progression

- Understand content from 4.NF.5 prior to beginning instruction on 4.NF.6. 4.NF.6.LP.A
- Explore place values using place value models. (pennies and dimes) 4.NF.6.LP.B
- Recognize pennies and dimes. 4.NF.6.LP.C
- Know the names and values of pennies and dimes. 4.NF.6.LP.D
- Know the symbols for dollars ($), cents (¢), and decimal point (.). 4.NF.6.LP.E
- Record the value of a collection of pennies using dollar or cent notation. 4.NF.6.LP.F
- Engagement Statements (demonstration of engaged in the topic) 4.NF.6.LP.G
- Interact with pennies and dimes. 4.NF.6.LP.H

7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record

the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. 4.NF.7

Complexity a

a Compare two decimals using place value models and the <, >, and = symbols (limit to tenths with tenths and hundredths to hundredths, includes whole numbers to tens). 4.NF.7A

Complexity b

b Compare two decimals using place value models and the <, >, and = symbols (limit to tenths with tenths and hundredths to hundredths, no whole numbers). 4.NF.7B

Complexity c

c Identify the tenths and hundredths place on a place value chart and in a given decimal using physical or visual representations. 4.NF.7C

Learning Progression

- Understand content from 4.NF.5-6 prior to beginning instruction on 4.NF.7. 4.NF.7.LP.A
- Explore place values using place value chart. 4.NF.7.LP.B
- Recognize the word names for tenths and hundredths. 4.NF.7.LP.C
- Know the symbol for decimal point (.). 4.NF.7.LP.D
- Understand the location of the decimal point on a place value chart. 4.NF.7.LP.E
- Recognize the value of a decimal in hundredths using a place value chart. 4.NF.7.LP.F
- Use language of tenths and hundredths in real-world contexts. 4.NF.7.LP.G
- Engagement Statements (demonstration of engaged in the topic) 4.NF.7.LP.H
- Interact with place value chart. 4.NF.7.LP.I

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a twocolumn table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, and 3 and 300. 4.MD.1

Complexity a

a Convert between km and m, m and cm, kg and g using place value charts or other physical/visual representations. 4.MD.1A

Complexity b

b Identify whether a measurement is “more than,” “less than,” or “same as” another metric measurement using a place value chart or other physical/visual models. For example, 1.5 kg is larger than 500 g. 4.MD.1B

Complexity c

c Determine the best metric unit to measure a specific real-world item (metric units include centimeters and meters, kilograms and grams, liters). 4.MD.1C

Learning Progression

- Describe measurable attributes of a single object using terms such as heavy/light, long/short, short/tall, etc. 4.MD.1.LP.A
- Explore measurements in cups and liters; inches, meters, and centimeters; grams and kilograms. 4.MD.1.LP.B
- Directly compare two amounts of liquid using measuring cups and liters to see which one holds more and which one holds less. 4.MD.1.LP.C
- Directly compare two beakers in liters to see which one holds more and which one holds less. 4.MD.1.LP.D
- Directly compare two distance measurements in centimeters, inches, and meters using rulers and meter sticks. 4.MD.1.LP.E
- Directly compare two measurements in grams and kilograms using different types of scales. 4.MD.1.LP.F
- Use different types of scales to measure mass of objects in kilograms and grams. 4.MD.1.LP.G
- Use different types of containers to measure volume of liquids in liters. 4.MD.1.LP.H
- Use different types of meter sticks and rulers to measure distance in meters and centimeters. 4.MD.1.LP.I
- Engagement Statements (demonstration of engaged in the topic) 4.MD.1.LP.J
- Interact with measurement tools for length, volume, and mass. 4.MD.1.LP.K

2 Solve realworld problems involving money, time, and metric measurement. a. Using models, add and subtract money and express the answer in decimal notation. b. Using number line diagrams, clocks, or other models, add and subtract intervals of time in hours and minutes. c. Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects. 4.MD.2

Complexity a

a1 Solve realworld problems involving addition or subtraction of coins and bills using visual and/or physical representations (limit amounts to less than $100). 4.MD.2A1

a2 Solve word problems involving addition and subtraction of time intervals in 15 minutes with visual and/or physical representations. 4.MD.2A2

a3 Solve real-world problems involving mass or volume by selecting appropriate operations with physical and/or visual representations. 4.MD.2A3

Complexity b

b1 Solve realworld problems with addition of collections of coins or bills with visual and/or physical representations (limit amounts to less than $50). 4.MD.2B1

b2 Solve word problems involving addition of time intervals of 30 minutes with visual and/or physical representations. 4.MD.2B2

b3 Solve real-world problems by measuring liquid volumes and masses of objects using standard units of measure with physical and/or visual representations. 4.MD.2B3

Complexity c

c1 Identify the value of all coins. Find the total of a collection of all pennies or all dimes or all nickels. 4.MD.2C1

c2 Solve word problems involving addition of time intervals of one hour with visual and/ or physical representation. 4.MD.2C2

c3 Solve real-world problems by selecting the appropriate tool to measure metric volume or mass with visual and physical representations. 4.MD.2C3

Learning Progression

- Understand content from 4.NF.5-7 before beginning instruction with pennies, nickels, and dimes. 4.MD.2.LP.A
- Understand content from 4.MD.2 before beginning instruction on 4.MD.3c. 4.MD.2.LP.B
- Explore place values using place value models. (pennies, nickels, and dimes) 4.MD.2.LP.C
- Recognize pennies, nickels, and dimes. 4.MD.2.LP.D
- Describe differences between U.S. coins. 4.MD.2.LP.E
- Know the names and values of pennies, nickels, and dimes. 4.MD.2.LP.F

- Know the symbols for dollars ($), cents (¢), and decimal point (.). 4.MD.2.LP.G
- Record the value of a collection of pennies, nickels, or dimes using dollar or cent notation. 4.MD.2.LP.H
- Relate counting to 12 to telling time on an analog clock. 4.MD.2.LP.I
- Tell time using a digital clock. 4.MD.2.LP.J
- Read the hour hand on an analog clock at different times of a day. 4.MD.2.LP.K
- Know the meaning of the hour and the minute hands on an analog clock. 4.MD.2.LP.L
- Use different types of scales to measure mass of objects in kilograms and grams. 4.MD.2.LP.M
- Use different types of containers to measure volume of liquids in liters. 4.MD.2.LP.N
- Engagement Statements (demonstration of engaged in the topic) 4.MD.2.LP.O
- Interact with pennies and dimes. 4.MD.2.LP.P
- Interact with analog clocks. 4.MD.2.LP.Q
- Interact with measurement tools for volume and mass. 4.MD.2.LP.R

3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter. 4.MD.3

Complexity a

a1 Given a rectangle with side lengths marked, find area (limit to whole number measurements). 4.MD.3A1

a2 Solve real-world problems involving perimeter. Find the perimeter or given the perimeter, find the missing side length (limit to whole numbers). 4.MD.3A2

Complexity b

b1 Find the area of rectangles by counting unit squares and understand that a square with a side length of 1 unit is called a “unit square.” 4.MD.3B1

b2 Find the perimeter of rectangles drawn on grid paper (whole numbers only). 4.MD.3B2

Complexity c

c1 Find the area of rectangles with whole-number side lengths by counting unit squares. 4.MD.3C1

c2 Find the perimeter of rectangles by counting the number of unit squares that fit around the shape. 4.MD.3C2

Learning Progression

- Identify surfaces where an area can be measured. 4.MD.3.LP.A
- Lay unit squares on a flat surface. 4.MD.3.LP.B
- Count unit squares on a flat surface. 4.MD.3.LP.C
- Arrange unit squares into rows and columns. 4.MD.3.LP.D
- Identify surfaces where a perimeter can be measured. 4.MD.3.LP.E
- Lay inch squares around a flat surface. 4.MD.3.LP.F
- Count inch squares all the way around a flat surface. 4.MD.3.LP.G
- Measure length and width with units and record the measurement. 4.MD.3.LP.H
- Engagement Statements (demonstration of engaged in the topic) 4.MD.3.LP.I
- Interact with flat two-dimensional surfaces. 4.MD.3.LP.J

Represent and interpret data.

4 Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade. 4.MD.4

Complexity a

a Interpret data from a given line, picture, or bar graph to solve a multi-step problem (limit to whole numbers). 4.MD.4A

Complexity b

b Interpret data represented in a graph by solving one-step “how many more” and “how many less” problems (limit to whole numbers). 4.MD.4B

Complexity c

c Given a bar or picture graph, build a graph based on student sorted data. For example, votes for 4 different candidates or weather types, or occurrences of event or behavior. 4.MD.4C

Learning Progression

- Classify objects into categories. 4.MD.4.LP.A
- Count the number of objects in each category. 4.MD.4.LP.B
- Create a bar graph or picture graph with a scale of 1 by stacking physical objects. 4.MD.4.LP.C
- Engagement Statements (demonstration of engaged in the topic) 4.MD.4.LP.D
- Interact with a bar graph or picture graph. 4.MD.4.LP.E

Geometric measurement: Understand concepts of angle and measure angles.

5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. a. Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees. 4.MD.5

Complexity a

a Label the parts of an angle (two rays, common endpoint) and identify, 45-, 90-, 180-, and 360-degree angles. 4.MD.5A

Complexity b

b Match words to the parts of an angle (two rays, common endpoint). 4.MD.5B

Complexity c

c Identify an angle in a given shape. 4.MD.5C

Learning Progression

- Explore angle measurement using a protractor or angle ruler. 4.MD.5.LP.A
- Recognize angles. 4.MD.5.LP.B
- Engagement Statements (demonstration of engaged in the topic) 4.MD.5.LP.C
- Interact with a protractor or angle ruler. 4.MD.5.LP.D

6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 4.MD.6

Complexity a

a Measure 45-, 90-, and 180-degree angles. 4.MD.6A

Complexity b

b Match measures and diagrams of 45-, 90-, and 180-degree angles. 4.MD.6B

Complexity c

c Given two measuring tools, select a protractor as a tool to measure and sketch angles. 4.MD.6C

Learning Progression

- Explore angle measurement using a protractor or angle ruler. 4.MD.6.LP.A
- Recognize angles. 4.MD.6.LP.B
- Engagement Statements (demonstration of engaged in the topic) 4.MD.6.LP.C
- Interact with a protractor or angle ruler. 4.MD.6.LP.D

7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures

of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. 4.MD.7

Complexity a

a Solve measurement problems involving angles using addition and subtraction. 4.MD.7A

Complexity b

b Solve measurement problems involving angles using addition. 4.MD.7B

Complexity c

c Solve addition measurement problems involving angles to show that two smaller angles make a larger angle. 4.MD.7C

Learning Progression

- Understand content from 4.MD.5-6 and 4.NBT.4 before beginning instruction 4.MD.7 4.MD.7.LP.A
- Explore angle measurement using an angle ruler. 4.MD.7.LP.B
- Recognize angles. 4.MD.7.LP.C
- Use two angle rulers to combine angles. 4.MD.7.LP.D
- Create a larger angle by combining two smaller angles. 4.MD.7.LP.E
- Find sums of two 2-digit whole numbers angle measures whose sums are within 90 without regrouping. 4.MD.7.LP.F
- Engagement Statements (demonstration of engaged in the topic) 4.MD.7.LP.G
- Interact with an angle ruler. 4.MD.7.LP.H

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1 Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in twodimensional figures. 4.G.1

Complexity a

a Match word to corresponding picture of perpendicular and parallel lines and angles (right, acute, obtuse). 4.G.1A

Complexity b

b Match word to corresponding picture of points, lines, line segments rays, and angles. 4.G.1B

Complexity c

c Identify a point, line, and a line segment. 4.G.1C

Learning Progression

- Engagement Statements (demonstration of engaged in the topic) 4.G.1.LP.A
- Interact with two-dimensional figures. 4.G.1.LP.B

2 Classify twodimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. 4.G.2

Complexity a

a Label a two-dimensional shape to show understanding of parallel or perpendicular lines. 4.G.2A

Complexity b

b Sort twodimensional shapes based on presence of parallel and/or perpendicular lines. 4.G.2B

Complexity c

c Match two-dimensional shapes in the environment based on parallel lines. 4.G.2C

Learning Progression

- Identify shapes as two-dimensional (lying in a plane, flat) or three-dimensional (solid). 4.G.2.LP.A
- Name shapes regardless of their orientations or overall size. 4.G.2.LP.B
- Recognize lines. 4.G.2.LP.C
- Understand parallel lines never intersect. 4.G.2.LP.D
- Engagement Statements (demonstration of engaged in the topic) 4.G.2.LP.E
- Interact with two- and three-dimensional objects in their environment. 4.G.2.LP.F